Notes Section 5.4

Determine if the given numbers could be the lengths of the sides of a right triangle.

1. $a = 9$, $b = 40$, $c = 41$
2. $a = \sqrt{6}$, $b = 6$, $c = 15$
3. $a = 18$, $b = 24$, $c = 30$
4. $a = 10$, $b = 20$, $c = 24$
5. $a = 8$, $b = 15$, $c = 17$
6. $a = 10$, $b = 28$, $c = 29$

$8^2 + 15^2 = 17^2$
$289 = 289$ ✓

$10^2 + 28^2 = 29^2$
$884 \neq 841$
NO

Warm Up

$a^2 + b^2 = c^2$
$9^2 + 40^2 = 41^2$
$1681 = 1681$ ✓

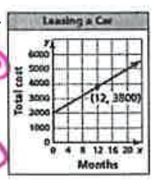
$(\sqrt{6})^2 + 6^2 = 15^2$
$42 \neq 225$ NO

$18^2 + 24^2 = 30^2$
$900 = 900$ ✓

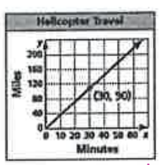
$10^2 + 20^2 = 24^2$ NO
$500 \neq 576$

Use the graph to write an equation of the line and interpret the slope.

1.



2.



(0, 2,000)
x_1 y_1
(12 ... 00)
x_2 y_2
y-Int = 2000

$m = \frac{3800-2000}{12-0}$ $\frac{1800}{12} = \frac{300}{2} = 150$

(0, 0)
(x_1, y_1)
(30, 30)
x_2 y_2
y-Int = 0

$m = \frac{30}{30} = 1$

Cumulative Warm Up

$y = 150x + 2000$ ← #1

$y = x$ ← #2

*begin by locating y-Intercept
*Calculate slope - pick two points

Essential Question

How can you solve a radical equation?

Essential Question

What you will learn:

*Solve equations containing radicals and rational exponents

*Solve radical Inequalities

Notes Section 5.4

Work with a partner. Match each radical equation with the graph of its related radical function. Explain your reasoning. Then use the graph to solve the equation, if possible. Check your solutions.

a. $\sqrt{x-1}-1=0$

$+1 \quad +1$

$(\sqrt{x-1})^2=(1)^2$

$x-1=1$

$x=2$

b. $\sqrt{2x+2}-\sqrt{x+4}=0$

$(\sqrt{2x+2})^2=(\sqrt{x+4})^2$

$2x+2=x+4$

$\boxed{x=2}$

c. $(\sqrt{9-x^2})^2=(0)^2$

$9-x^2=0$

$-1(x^2-9)=0$

$-1(x-3)(x+3)=0$

$(x-3)=0 \qquad x+3=0$

$x=3 \qquad x=-3$

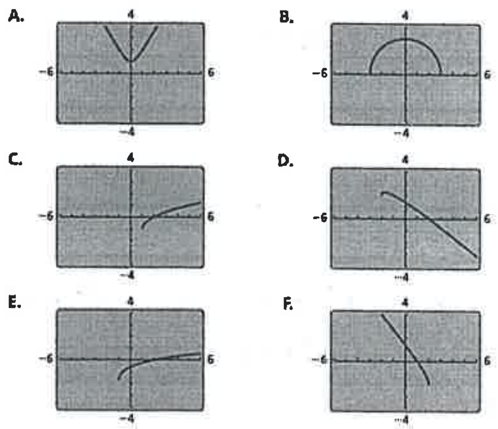
d. $\sqrt{x+2}-x=0$

e. $\sqrt{-x+2}-x=0$

f. $\sqrt{3x^2+1}=0$

* d, e, f student practice

Exploration 1



each equation from previous slide is represented in graph form. Plug each equation into graphing software to match graphs.

Exploration 1B

Work with a partner. Look back at the radical equations in Exploration 1. Suppose that you did not know how to solve the equations using a graphical approach.

a. Show how you could use a *numerical approach* to solve one of the equations. For instance, you might use a spreadsheet to create a table of values.

b. Show how you could use an *analytical approach* to solve one of the equations. For instance, look at the similarities between the equations in Exploration 1. What first step may be necessary so you could square each side to eliminate the radical(s)? How would you proceed to find the solution?

discussion points

Exploration 2

Core Concept

Solving Radical Equations

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the resulting equation using techniques you learned in previous chapters. Check your solution.

Core Concept

Radical equations are solved in the same steps as absolute value equations.

Solve (a) $2\sqrt{x+1}=4$ and (b) $\sqrt[3]{2x-9}-1=2$.

$\overline{2}\quad\overline{2}$

$\sqrt{x+1}=2$

$(\sqrt{x+1})^2=(2)^2$

$x+1=4$

$x=3$

$\sqrt[3]{2x-9}=3$

$(\sqrt[3]{2x-9})^3=(3)^3$

$2x-9=27$

$+9\quad+9$

$\frac{2x}{2}=\frac{36}{2}$

$x=18$

Example 1

* Isolate radical expression
  * undo addition or subtraction first
  * undo multiplication or division
* using the index, raise both sides to that power
* solve for the variable

* always check answers.

Solve the equation. Check your solution.

1. $\sqrt[3]{x}-9=-6$ **2.** $\sqrt{x+25}=2$ **3.** $2\sqrt[3]{x-3}=4$

Monitoring Progress 1-3

* student practice

In a hurricane, the mean sustained wind velocity v (in meters per second) can be modeled by $v(p)=6.3\sqrt{1013-p}$, where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of the hurricane when the mean sustained wind velocity is 54.5 meters per second.

$V(p) = 6.3\sqrt{1013-p}$

$\frac{54.5}{6.3} = \frac{6.3\sqrt{1013-p}}{6.3}$

$(8.65)^2 \approx (\sqrt{1013-p})^2$

$74.8 \approx 1013 - p$

$-938.2 = -p$

$938.2 = p$

Example 2

* real world situation

Air pressure at the center of a hurricane is about 938 milibars

Change = to ≈ when you round values.

4. WHAT IF? Estimate the air pressure at the center of the hurricane when the mean sustained wind velocity is 48.3 meters per second.

Monitoring Progress 4

* change the value of V(p) to 48.3 and resolve.

Solve $x+1=\sqrt{7x+15}$.

$(x+1)^2 = (\sqrt{7x+15})^2$

$(x+1)(x+1) = 7x+15$

$x^2+2x+1 = 7x+15$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x-7=0 \quad x+2=0$

$x=7 \quad x=-2$

Example 3

Raising each side of an equation to the same exponent may introduce solutions that are not solutions or are extraneous solutions. You should always check each solution back in the original equation and eliminate any solution that does not work.

Solve $\sqrt{x+2}+1=\sqrt{3-x}$.

$(\sqrt{x+2}+1)^2=(\sqrt{3-x})^2$

$x+2+2\sqrt{x+2}+1=3-x$

$x+3+2\sqrt{x+2}=3-x$

$-x\ -3 \qquad -3\ -x$

$\frac{2\sqrt{x+2}}{2}=\frac{-2x}{2}$

$\sqrt{x+2}=-x$

$(\sqrt{x+2})^2=(-x)^2$

Example 4

(Cont.) $x+2=x^2$

$-x\ -2 \qquad -x-2$

$0=x^2-x-2$

$0=(x-2)(x+1)$

$x-2=0 \qquad x+1=0$

$x=2 \qquad x=-1$

$\sqrt{2+2}+1=\sqrt{3-2} \qquad \sqrt{-1+2}+1=\sqrt{3-(-1)}$

$\sqrt{4}+1=\sqrt{1} \qquad \sqrt{1}+1=\sqrt{4}$

$3\neq 1 \qquad 2=2$ ✓

Solve the equation. Check your solution(s).

5. $\sqrt{10x+9}=x+3$ **6.** $\sqrt{2x+5}=\sqrt{x+7}$ **7.** $\sqrt{x+6}-2=\sqrt{x-2}$

Monitoring Progress 5-7

* Student practice

Solve $(2x)^{3/4}+2=10$.

$-2\ -2$

$(2x)^{3/4}=8$

$\left[(2x)^{3/4}\right]^{4/3}=(8)^{4/3}$

$2x=(2^3)^{4/3}$

$2x=16$

$x=8$

Example 5

To solve for rational exponents:

* Isolate the term with the rational exponent

* raise each side of the equation to the reciprocal of the rational exponent.

Example $\frac{3}{4}$ reciprocal $\frac{4}{3}$

Solve $(x+30)^{1/2} = x$.

$$\left[(x+30)^{1/2}\right]^2 = x^2$$
$$x+30 = x^2$$
$$0 = x^2 - x - 30$$
$$0 = (x-6)(x+5)$$
$$x-6=0 \qquad x+5=0$$
$$x=6 \qquad x=-5$$

Example 6

* always check work

* you could also change the rational exponents into radicals and solve the same way. Either method will work.

Solve the equation. Check your solution(s).

8. $(3x)^{1/3} = -3$ **9.** $(x+6)^{1/2} = x$ **10.** $(x+2)^{3/4} = 8$

Monitoring Progress 8-10

* Student practice

Solve $3\sqrt{x-1} \le 12$.

$$\frac{3\sqrt{x-1}}{3} \le \frac{12}{3}$$
$$\sqrt{x-1} \le 4$$
$$(\sqrt{x-1})^2 \le (4)^2$$
$$x-1 \le 16$$
$$+1 \qquad +1$$
$$x \le 17$$

Example 7

* Solving inequalities work the same way as equations. Follow the same steps.

Notes Section 5.4

11. Solve (a) $2\sqrt{x}-3\geq 3$ and (b) $4\sqrt[3]{x+1}<8$.

$+3 \quad +3$

$\frac{2\sqrt{x}}{2} \geq \frac{6}{2}$

$\sqrt{x} \geq 3$

$(\sqrt{x})^2 \geq (3)^2$

$x \geq 9$

$\frac{4\sqrt[3]{x+1}}{4} < \frac{8}{4}$

$(\sqrt[3]{x+1})^3 < (2)^3$

$x+1 < 8$

$-1 \quad -1$

$x < 7$

Monitoring Progress 11

* remember rules about multiplying and dividing with negatives and how it affects the Inequality sign.

• Writing Prompt: It is necessary to check apparent solutions when solving radical equations because …

Closure

* Reflect on the lesson are there any questions that still remain?