Factor the expression.

1. $25z^2 - y2$

$(5z-y)(5z+y)$

2. $25x^2 - 1$

$(5x-1)(5x+1)$

3. $49x^2 + 28xy + 4y^2$

$(7x+2y)(7x+2y)$

4. $\frac{1}{x^2} - 1$

$x^2(\frac{1}{x^2} - 1)$

$1 - x^2$

$-1(x^2-1)$

$-1(x-1)(x+1)$

5. $8y^2 - 2$

$2(4y^2-1)$

$2(2y-1)(2y+1)$

6. $4rs^2 - 4rs + r$

$r(4s^2 - 4s + 1)$

$r(2s-1)(2s-1)$

Warm Up

#1 + 2: difference of perfect squares.

#3: perfect square trinomial

#4+5: difference of perfect squares (after taking a GCF)

#6: perfect square trinomial (after taking a GCF)

Identify the function family and describe the domain and range.

1. $g(x) = |x - 3|$

absolute value

2. $g(x) = 4x - 3$

Linear

3. $f(x) = 6x^2 + 1$

quadratic

4. $h(x) = |x + 4| - 1$

absolute value

5. $f(x) = -3x - 10$

linear

6. $f(x) = -x^2 - 5$

quadratic

Cumulative Warm Up

*Use graphing software to show domain and range.

Essential Question

How can you complete the square for a quadratic expression?

Essential Question

What you will learn?

- Solve quadratic equations using square roots.
- Solve quadratic equations by completing the square
- Write quadratic functions in vertex form.

~~**Work with a partner.** Use algebra tiles to complete the square for the expression $x^2 + 6x$.~~

~~**a.** You can model $x^2 + 6x$ using one x^2-tile and six x-tiles. Arrange the tiles in a square. Your arrangement will be incomplete in one of the corners.~~

~~**b.** How many 1-tiles do you need to complete the square?~~

~~**c.** Find the value of c so that the expression $x^2 + 6x + c$ is a perfect square trinomial.~~

~~**d.** Write the expression in part (c) as the square of a binomial.~~

Exploration 1

Skip

~~**Work with a partner.**~~

~~**a.** Use the method outlined in Exploration 1 to complete the table.~~

Expression	Value of c needed to complete the square	Expression written as a binomial squared
$x^2 + 2x + c$		
$x^2 + 4x + c$		
$x^2 + 8x + c$		
$x^2 + 10x + c$		

~~**b.** Look for patterns in the last column of the table. Consider the general statement $x^2 + bx + c = (x + d)^2$. How are d and b related in each case? How are c and d related in each case?~~

~~**c.** How can you obtain the values in the second column directly from the coefficients of x in the first column?~~

Exploration 2

Skip

Solve $x^2 - 16x + 64 = 100$ using square roots.

$(x-8)(x-8) = 100$

$(x-8)^2 = 100$

$\sqrt{(x-8)^2} = \pm\sqrt{100}$

$x - 8 = \pm 10$

$x - 8 = 10$ $\quad$ $x - 8 = -10$

$x = 18$ $\quad$ $x = -2$

Example 1

$x^2 - 16x + 64 = 100$

$x \cdot x$ $\quad$ $8 \cdot 8$

$8 \cdot x$

$\times\ 2$

$16x$

perfect square trinomial

Solve the equation using square roots. Check your solution(s).

1. $x^2 + 4x + 4 = 36$ **2.** $x^2 - 6x + 9 = 1$ **3.** $x^2 - 22x + 121 = 81$

$(x+2)(x+2)=36$

$(x+2)^2=36$

$\sqrt{(x+2)^2}=\pm\sqrt{36}$

$x+2=\pm 6$

$x+2=6$

$x=4$

$x+2=-6$

$x=-8$

$(x-3)(x-3)=1$

$(x-3)^2=1$

$\sqrt{(x-3)^2}=\pm\sqrt{1}$

$x-3=\pm 1$

$x-3=1$ $x-3=-1$

$x=4$ $x=2$

Monitoring Progress 1-3

3.) $x^2-22x+121=81$

$(x-11)(x-11)=81$

$(x-11)^2=81$

$\sqrt{(x-11)^2}=\pm\sqrt{81}$

$x-11=\pm 9$

$x-11=9$ $x-11=-9$

$+11$ $+11$ $+11$ $+11$

$x=20$ $x=2$

Core Concept

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

Diagrams In each diagram, the combined area of the shaded regions is $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Core Concept

* what do you do if ax^2+bx+c is not a perfect square trinomial?

Some times we have to add a term to ax^2+bx to make it a perfect square trinomial.

Find the value of c that makes $x^2 + 14x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

$\frac{14}{2}=7$

$7^2=49$

$x^2+14x+49$

$(x+7)^2$

Example 2

- take the coefficient from the bx term
- divide the coefficient by 2
- take the quotient and raise it to the power of 2
- add this value to both sides.

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

4. $x^2 + 8x + c$

$\frac{8}{2} = 4$

$4^2 = 16$

$x^2 + 8x + 16$

$(x+4)^2$

5. $x^2 - 2x + c$

$\frac{2}{2} = 1$

$1^2 = 1$

$x^2 - 2x + 1$

$(x+1)^2$

6. $x^2 - 9x + c$

$\frac{9}{2}$

$\left(\frac{9}{2}\right)^2$

$\frac{81}{4}$ $\left(x+\frac{9}{2}\right)^2$

$x^2 - 9x + \frac{81}{4}$

Monitoring Progress 4-6

* perfect square trinomial can only be in the form of:

$ax^2 + bx + c = (\ + \ )(\ + \ )$

or

$ax^2 - bx + c = (\ - \ )(\ - \ )$

* the terms will always be square of ax^2 term and square of c term.

Solve $x^2 - 10x + 7 = 0$ by completing the square.

$\frac{10}{2} = 5 \quad 5^2 = 25$

$x^2 - 10x + 7 = 0$

$\quad\quad -7 \quad -7$

$x^2 - 10x + 25 = -7 + 25$

$(x-5)^2 = 18$

$\sqrt{(x-5)^2} = \pm\sqrt{18} < \begin{matrix}2\\9\end{matrix}$

$x - 5 = \pm 3\sqrt{2}$

$x - 5 = \pm 3\sqrt{2}$

$+5 \quad +5$

$x = 5 \pm 3\sqrt{2}$

Example 3

Solve $3x^2 + 12x + 15 = 0$ by completing the square.

$\frac{3x^2}{3} + \frac{12x}{3} + \frac{15}{3} = \frac{0}{3}$

$x^2 + 4x + 5 = 0$

$\quad -5 \quad -5$

$x^2 + 4x = -5$

$\frac{4}{2} = 2$

$2^2 = 4$

$x^2 + 4x + 4 = -5 + 4$

$(x+2)^2 = -1$

$\sqrt{(x+2)^2} = \pm\sqrt{-1}$

$x + 2 = \pm i$

$x + 2 = i$

$-2 \quad -2$

$x = -2 + i$

$x + 2 = -i$

$-2 \quad -2$

$x = -2 - i$

Example 4

- when there is a GCF - begin by dividing every term on both sides of equal sign by GCF

- remember rules of imaginary unit.

Solve the equation by completing the square.

7. $x^2 - 4x + 8 = 0$ **8.** $x^2 + 8x - 5 = 0$ **9.** $-3x^2 - 18x - 6 = 0$

$x^2 - 4x = -8$

$x^2 - 4x + 4 = -4$

10. $4x^2 + 32x = -68$ **11.** $6x(x + 2) = -42$ **12.** $2x(x - 2) = 200$

Monitoring Progress 7-12

Student practice

Write $y = x^2 - 12x + 18$ in vertex form. Then identify the vertex.

$\frac{12}{2} = 6$

$6^2 = 36$

$y = x^2 - 12x + 18$ function

$y + ? = (x^2 - 12x + ?) + 18$

$y + 36 = (x^2 - 12x + 36) + 18$

$y + 36 = (x - 6)^2 + 18$

$-36 \qquad -36$

$y = (x - 6)^2 - 18$

The vertex = $(6, -18)$

Example 5

Vertex form ⇒ $y = a(x - h)^2 + k$ where (h, k) is the vertex of the graph of the function.

(h, k)

remember $(x - h)^2$ always use opposite

Write the quadratic function in vertex form. Then identify the vertex.

13. $y = x^2 - 8x + 18$ **14.** $y = x^2 + 6x + 4$ **15.** $y = x^2 - 2x - 6$

$\frac{8}{2} = 4$

$4^2 = 16$

$y + ? = (x^2 - 8x \quad) + 18$

$y + 16 = (x^2 - 8x + 16) + 18$

$y + 16 = (x - 4)^2 + 18$

$-16 \qquad -16$

$y = (x - 4)^2 + 2$

V: $(4, 2)$

Monitoring Progress 13-15

Students practice #14 and 15

The height y (in feet) of a baseball t seconds after it is hit can be modeled by the function $y = -16t^2 + 96t + 3$. Find the maximum height of the baseball. How long does the ball take to hit the ground?

$\frac{6}{2} = 3$

$3^2 = 9$

$y = -16t^2 + 96t + 3$

$y = -16(t^2 - 6t) + 3$

$y + ? = -16(t^2 - 6t + ?) + 3$

$y + (-16)(9) = -16(t^2 - 6t + 9) + 3$

$y - 144 = -16(t-3)^2 + 3$

$+144 \qquad +144$

$y = -16(t-3)^2 + 147$

Example 6

Vertex $= (3, 147)$

Can also get vertex through axis of symmetry

$0 = -16(t-3)^2 + 147$

$-147 \qquad -147$

$\frac{-147}{-16} = \frac{-16(t-3)^2}{-16}$

$9.1875 = (t-3)^2$

$\pm\sqrt{9.1875} = \sqrt{(t-3)^2}$

$\pm\sqrt{9.1875} = t - 3$

$+3 \qquad +3$

$3 \pm \sqrt{9.1875} = t$

$3 + \sqrt{9.1875} = t \qquad 3 - \sqrt{9.1875}$

≈ 6 sec. $\qquad -.03$ sec

time can't be negative

16. WHAT IF? The height of the baseball can be modeled by $y = -16t^2 + 80t + 2$. Find the maximum height of the baseball. How long does the ball take to hit the ground?

*we will learn a quicker way to solve this type of question when we do quadratic equations.

Monitoring Progress 16

Writing Prompt: To solve a quadratic equation by completing the square you ...

Closure