Determine the value of x in the expression.

1. $2^x = 4$ 2. $8^x = 1$ 3. $\left(\frac{2}{3}\right)^x = \frac{8}{27}$

$2^x = 2^2$ $x=0$ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3$

$x = 2$ $x = 3$

4. $4^x = \frac{1}{4}$ 5. $4^x = 2$ 6. $4^x = \frac{1}{2}$

$4^x = 4^{-1}$ $(2^2)^x = 2^1$ $(2^2)^x = 2^{-1}$

$x = -1$ $2x = 1$ $2x = -1$

$x = \frac{1}{2}$ $x = -\frac{1}{2}$

Warm Up

* review of exponents

* remember when the base numbers are equal the exponents are equal

* review of how to make exponents positive or negative.

Use the parent function to sketch the graph of each function.

1. Parent function: $y = x^2$

a. $y = x^2 - 2$ b. $y = (x + 3)^2$ c. $y = -(x - 1)^2$

2. Parent function: $y = \sqrt{x}$

a. $y = 2\sqrt{x}$ b. $y = \sqrt{-x}$ c. $y = -\sqrt{x - \frac{1}{2}} + 3$

Cumulative Warm Up

Skip

· Understand basic parent function - how do the operations change the parent function.

Essential Question

What are some of the characteristics of the graph of a logarithmic function?

Essential Question

What you will learn:

· Define and evaluate logarithms

· Use Inverse properties of logarithms and exponential functions

· Graph logarithm functions

Work with a partner. Find the value of x in each exponential equation. Explain your reasoning. Then use the value of x to rewrite the exponential equation in its equivalent logarithmic form, $x = \log_b y$.

a. $2^x = 8$ **b.** $3^x = 9$ **c.** $4^x = 2$

d. $5^x = 1$ **e.** $5^x = \frac{1}{5}$ **f.** $8^x = 4$

Exploration 1

Skip

Work with a partner. Complete each table for the given exponential function. Use the results to complete the table for the given logarithmic function. Explain your reasoning. Then sketch the graphs of f and g in the same coordinate plane.

a.

x	-2	-1	0	1	2
$f(x) = 2^x$					

x					
$g(x) = \log_2 x$	-2	-1	0	1	2

b.

x	-2	-1	0	1	2
$f(x) = 10^x$					

x					
$g(x) = \log_{10} x$	-2	-1	0	1	2

Exploration 2

Skip

Work with a partner. Use the graphs you sketched in Exploration 2 to determine the domain, range, x-intercept, and asymptote of the graph of $g(x) = \log_b x$, where b is a positive real number other than 1. Explain your reasoning.

Exploration 3

Skip

Core Concept

Definition of Logarithm with Base b

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \quad \text{if and only if} \quad b^x = y.$$

The expression $\log_b y$ is read as "log base b of y."

Core Concept

$\log_b y = x$

log base answer = exponent

$9^2 = 81$ — exponent, base, answer

$\log_9 81 = 2$

Rewrite each equation in exponential form.

a. $\log_2 16 = 4$ **b.** $\log_4 1 = 0$

$2^4 = 16$ $4^0 = 1$

c. $\log_{12} 12 = 1$ **d.** $\log_{1/4} 4 = -1$

$12^1 = 12$ $\left(\frac{1}{4}\right)^{-1} = 4$

Example 1

* Student practice

Rewrite each equation in logarithmic form.

a. $5^2 = 25$ **b.** $10^{-1} = 0.1$

$\log_5 25 = 2$ $\log_{10} .1 = -1$

c. $8^{2/3} = 4$ **d.** $6^{-3} = \frac{1}{216}$

$\log_8 4 = \frac{2}{3}$ $\log_6 \frac{1}{216} = -3$

Example 2

* Student practice

Evaluate each logarithm.

a. $\log_4 64$ **b.** $\log_5 0.2$ **c.** $\log_{1/5} 125$ **d.** $\log_{36} 6$

$4^x = 64$ $\quad 5^x = .2$ $\quad \frac{1}{5}^x = 125$

$4^x = 4^3$ $\quad 5^x = \frac{2}{10} = \frac{1}{5}$ $\quad 5^{-x} = 5^3$

$x = 3$ $\quad 5^x = 5^{-1}$ $\quad -x = 3$

$x = -3$

$x = -1$

$36^x = 6$

$(6^2)^x = 6$

$6^{2x} = 6^1$

$2x = 1$

$x = \frac{1}{2}$

Example 3

Evaluate (a) log 8 and (b) ln 0.3 using a calculator. Round your answer to three decimal places.

use graphing Calculator:

$\log 8 \approx 0.903$

$\ln 0.3 \approx -1.204$

Example 4

$\log_{10} x = \log x$

Common logarithm is a logarithm with a base 10.

natural logarithm is a logarithm with a base e

$\log_e$ or $\ln$

$\log_e x = \ln x$

Rewrite the equation in exponential form.

1. $\log_3 81 = 4$ **2.** $\log_7 7 = 1$

3. $\log_{14} 1 = 0$ **4.** $\log_{1/2} 32 = -5$

Rewrite the equation in logarithmic form.

5. $7^2 = 49$ **6.** $50^0 = 1$

7. $4^{-1} = \frac{1}{4}$ **8.** $256^{1/8} = 2$

Evaluate the logarithm. If necessary, use a calculator and round your answer to three decimal places.

9. $\log_2 32$ **10.** $\log_{27} 3$ **11.** $\log 12$ **12.** $\ln 0.75$

Monitoring Progress 1-12

*Student practice

Simplify (a) $10^{\log 4}$ and (b) $\log_5 25^x$.

$10^{\log 4}$

$10^{\log 4} = 4$

$b^{\log_b x} = x$

$\log_5 25^x =$

$= \log_5 (5^2)^x$

$= \log_5 (5^{2x})$

$= 2x$

$\log_b b^x = x$

Example 5

Using Inverse properties

by definition of a log, it follows that the log function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$

Find the inverse of each function.

a. $f(x) = 6^x$ **b.** $y = \ln(x + 3)$

$g(x) = \log_6 x$

$x = \ln(y + 3)$

$e^x = y + 3$

$e^x - 3 = y$

Example 6

Simplify the expression.

13. $8^{\log_8 x}$ **14.** $\log_7 7^{-3x}$

15. $\log_2 64^x$ **16.** $e^{\ln 20}$

17. Find the inverse of $y = 4^x$. **18.** Find the inverse of $y = \ln(x - 5)$.

Monitoring Progress 13-18

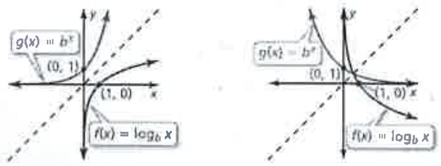
* Student practice

Core Concept

Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$ **Graph of $f(x) = \log_b x$ for $0 < b < 1$**



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

Core Concept

· this shows how the graph of a logarithm is the Inverse of an exponential function

· Notice the change of the asymptotes from the x-axis to the y-axis

Graph $f(x) = \log_3 x$.

Example 7

· use graphing calculator to graph

· Table function for values

Graph the function.

19. $y = \log_2 x$ **20.** $f(x) = \log_5 x$ **21.** $y = \log_{1/2} x$

Monitoring Progress 19-21

* Student practice

• **Response Logs:** "I got stuck with …" or "I'm not sure …" or "I made progress with. …"

Closure