**Graph the point and its reflection.**

**1.** point (5, 2) reflected in the $x$-axis

**2.** point (−1, 0) reflected in the $y$-axis

**3.** point (1, 2) reflected in the $y$-axis

Warm Up 1-3

have students use graph paper to do this

#1-6

Review what is a reflection?

reflection: a transformation that flips a graph over the line of reflection.

**Graph the point and its reflection.**

**4.** point (3, −3) reflected in the $x$-axis

**5.** point (−3, 3) reflected in the line through (−5, 1) and (−2, 1)

**6.** point (−4, 3) reflected in the line through (2, 2) and (4, 2)

Warm Up 4-6

**Determine the number of lines of symmetry, if any, for the letter. Draw each line of symmetry.**

1. T
2. E
3. L
4. W
5. H
6. R

Cumulative Warm Up

line of symmetry: divides a figure into two mirror-image halves.

## Essential Question

How do the graphs of $y = f(x) + k$, $y = f(x - h)$, and $y = -f(x)$ compare to the graph of the parent function $f$?

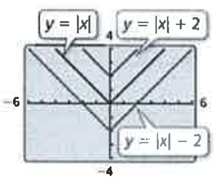
What you will learn:

- Write functions representing translations and reflections
- Write functions representing stretches and shrinks
- Write functions representing combinations of transformations

Essential Question

Use Student Journals pages: 7 - 8

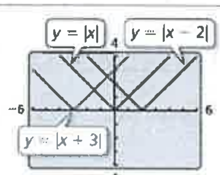
**Work with a partner.** Compare the graph of the function $y = |x| + k$ to the graph of the parent function $f(x) = |x|$.




Exploration 1

- If k is positive, the graph is shifted up k units.
- If k is negative, the graph is shifted down |k| units
- Why would we use absolute value when talking about the negative shift?
  - measure of distance always has to be positive.

**Work with a partner.** Compare the graph of the function $y = |x - h|$ to the graph of the parent function $f(x) = |x|$.




Exploration 2

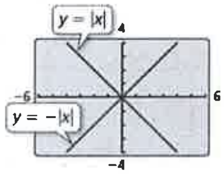
- If h is positive, the graph is shifted right h units.
- If h is negative, the graph is shifted left |h| units.

**Work with a partner.** Compare the graph of the function

$y = -|x|$

to the graph of the parent function

$f(x) = |x|$.



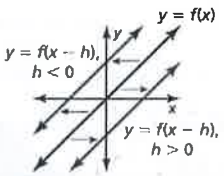

Exploration 3

· The graph is reflected in the x-axis.

## Core Concept

### Horizontal Translations

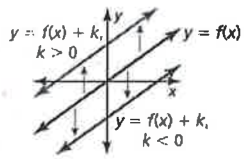
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.




Subtracting $h$ from the **inputs** before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

### Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.




Adding $k$ to the **outputs** shifts the graph down when $k < 0$ and up when $k > 0$.

Core Concept

Can do some examples with either the graphing Calculator or Desmos to show the translations.

Let $f(x) = 2x + 1$.

**a.** Write a function $g$ whose graph is a translation 3 units down of the graph of $f$.

**b.** Write a function $h$ whose graph is a translation 2 units to the left of the graph of $f$.

a) translation 3 units down is a vertical translation that adds (-3) to each output.

b) A translation 2 units to the left is a horizontal translation that subtracts (-2) from each input value.

Example 1

a.) $g(x) = f(x) + (-3)$
$= 2x + 1 + (-3)$
$= 2x - 2$

b.) $h(x) = f(x - (-2))$
$= f(x+2)$
$= 2(x+2) + 1$
$= 2x + 5$

## Core Concept

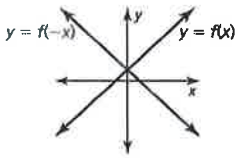
**Reflections in the $x$-axis**

The graph of $y = -f(x)$ is a reflection in the $x$-axis of the graph of $y = f(x)$.


Multiplying the outputs by $-1$ changes their signs.

**Reflections in the $y$-axis**

The graph of $y = f(-x)$ is a reflection in the $y$-axis of the graph of $y = f(x)$.




Multiplying the inputs by $-1$ changes their signs.

Core Concept

· when you reflect a function in a line, the graphs are symmetric about the line.

Input values: from the domain
- x . values

Output values: from the range
- y - values.

Let $f(x) = |x + 3| + 1$.

**a.** Write a function $g$ whose graph is a reflection in the $x$-axis of the graph of $f$.

**b.** Write a function $h$ whose graph is a reflection in the $y$-axis of the graph of $f$.

b.) A reflection in the y- axis changes the sign of each input value.

$h(x) = f(-x)$

$= |-x+3| + 1$

$= |-1| \cdot |x-3| + 1$

$= |x-3| + 1$

Example 2

a.) A reflection in the x- axis changes the sign of each output value.

$g(x) = -f(x)$

$= -(|x+3| + 1)$

$= -|x+3| - 1$

**Write a function $g$ whose graph represents the indicated transformation of the graph of $f$. Use a graphing calculator to check your answer.**

**1.** $f(x) = 3x$; translation 5 units up

$g(x) = 3x+5$

**2.** $f(x) = |x| - 3$; translation 4 units to the right

$g(x) = |x-4| - 3$

**3.** $f(x) = -|x + 2| - 1$; reflection in the $x$-axis

$g(x) = |x+2| + 1$

**4.** $f(x)=\frac{1}{2}x+1$; reflection in the $y$-axis

$g(x) = -\frac{1}{2}x + 1$

Monitoring Progress 1-4

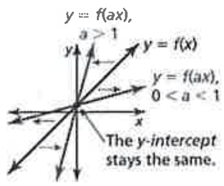
* Student practice *

## Core Concept

**Horizontal Stretches and Shrinks**

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

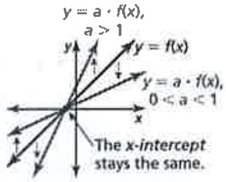
Multiplying the inputs by $a$ before evaluating the function stretches the graph horizontally (away from the $y$-axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the $y$-axis) when $a > 1$.




**Vertical Stretches and Shrinks**

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of $a$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the outputs by $a$ stretches the graph vertically (away from the $x$-axis) when $a > 1$, and shrinks the graph vertically (toward the $x$-axis) when $0 < a < 1$.




Core Concept

· The graphs of $y = f(-ax)$ and $y = -a(f(x))$ represent a stretch or shrink and a reflection in the x-or y-axis of the graph of $y = f(x)$.

Let $f(x) = |x - 3| - 5$. Write (a) a function $g$ whose graph is a horizontal shrink of the graph of $f$ by a factor of $\frac{1}{3}$, and (b) a function $h$ whose graph is a vertical stretch of the graph of $f$ by a factor of 2.

a) a horizontal shrink by a factor of $\frac{1}{3}$ multiplies each input value by 3.

$g(x) = f(3x)$

$= |3x - 3| - 5$

Example 3

b) A vertical stretch by a factor of 2 multiplies each output value by 2.

$h(x) = 2 \cdot f(x)$

$= 2 \cdot (|x - 3| - 5)$

$= 2|x - 3| - 10$

Write a function $g$ whose graph represents the indicated transformation of the graph of $f$. Use a graphing calculator to check your answer.

**5.** $f(x) = 4x + 2$; horizontal stretch by a factor of 2

$g(x) = 2x + 2$

**6.** $f(x) = |x| - 3$; vertical shrink by a factor of $\frac{1}{3}$

$g(x) = \frac{1}{3}x - 1$

Monitoring Progress 5-6

*Student Practice *

Let the graph of $g$ be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of $f(x) = x$. Write a rule for $g$.

Step 2: Then write a function g that represents the translation of h.

$g(x) = h(x) + 3$

$= 0.25x + 3$

Combinations of Transformations

Step 1: first write a function h that represents the vertical shrink of f.

$h(x) = .25 \cdot f(x)$

$= .25x$

Example 4

· .9 = vertical shrink

50 = translation down

You design a computer game. Your revenue for $x$ downloads is given by $f(x) = 2x$. Your profit is \$50 less than 90% of the revenue for $x$ downloads. Describe how to transform the graph of $f$ to model the profit. What is your profit for 100 downloads?

1.) Understand: given a function that represents revenue and a verbal statement the reps. profit. Find profit 100 downloads.

2.) Make plan: Function p reps profit, use function to find 100 downloads.

profit = 90% · revenue − 50

$p(x) = .9 \cdot f(x) - 50$

$= .9 \cdot 2x - 50$

$= 1.8x - 50$

Find profit 100 downloads

$p(100) = 1.8(100) - 50$

$= \$130$

Does it make sense?

Example 5

**7.** Let the graph of $g$ be a translation 6 units down followed by a reflection in the $x$-axis of the graph of $f(x) = |x|$. Write a rule for $g$. Use a graphing calculator to check your answer.

$g(x) = -|x| + 6$

**8.** WHAT IF? In Example 5, your revenue function is $f(x) = 3x$. How does this affect your profit for 100 downloads?

Profit = \$220

* Student practice *

· answers provided to check work.

Monitoring Progress 7-8

Describe the graph $g(x) = \frac{1}{2}|x-2|+3$ as compared to the graph of the parent function $f(x) = |x|$.

$f(x) = |x|$ ← parent function

$f(x) = |x-2|$ ← transformation 2 units right

$f(x) = |x-2| + 3$ ← transformation up 3 units.

$f(x) = \frac{1}{2}|x-2| + 3$

vertical shrink by $\frac{1}{2}$ toward the x-axis.

* can be used as an exit ticket *

Closure

Jun 12-9:53 AM