# 2.1 Exercises

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## Vocabulary and Core Concept Check

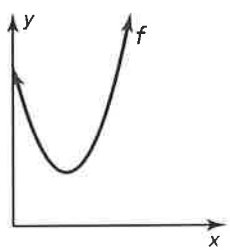
1. **COMPLETE THE SENTENCE** The graph of a quadratic function is called a(n) ________.
2. **VOCABULARY** Identify the vertex of the parabola given by $f(x) = (x + 2)^2 - 4$.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–12, describe the transformation of $f(x) = x^2$ represented by $g$. Then graph each function.** ***(See Example 1.)***

3. $g(x) = x^2 - 3$
4. $g(x) = x^2 + 1$
5. $g(x) = (x + 2)^2$
6. $g(x) = (x - 4)^2$
7. $g(x) = (x - 1)^2$
8. $g(x) = (x + 3)^2$
9. $g(x) = (x + 6)^2 - 2$
10. $g(x) = (x - 9)^2 + 5$
11. $g(x) = (x - 7)^2 + 1$
12. $g(x) = (x + 10)^2 - 3$

**ANALYZING RELATIONSHIPS** **In Exercises 13–16, match the function with the correct transformation of the graph of $f$. Explain your reasoning.**




13. $y = f(x - 1)$
14. $y = f(x) + 1$
15. $y = f(x - 1) + 1$
16. $y = f(x + 1) - 1$

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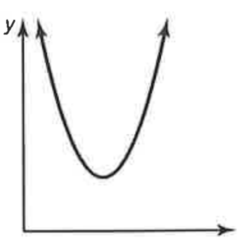


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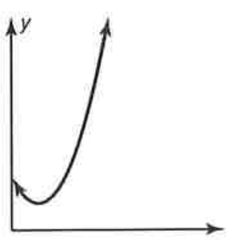


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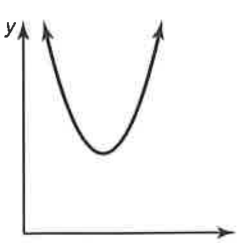


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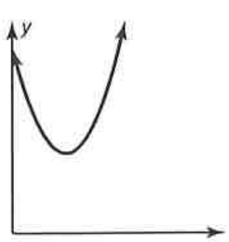


**In Exercises 17–24, describe the transformation of $f(x) = x^2$ represented by $g$. Then graph each function.** ***(See Example 2.)***

17. $g(x) = -x^2$
18. $g(x) = (-x)^2$
19. $g(x) = 3x^2$
20. $g(x) = \frac{1}{3}x^2$
21. $g(x) = (2x)^2$
22. $g(x) = -(2x)^2$
23. $g(x) = \frac{1}{5}x^2 - 4$
24. $g(x) = \frac{1}{2}(x - 1)^2$

**ERROR ANALYSIS** **In Exercises 25 and 26, describe and correct the error in analyzing the graph of $f(x) = -6x^2 + 4$.**

25. The graph is a reflection in the $y$-axis and a vertical stretch by a factor of 6, followed by a translation 4 units up of the graph of the parent quadratic function.

26. The graph is a translation 4 units down, followed by a vertical stretch by a factor of 6 and a reflection in the $x$-axis of the graph of the parent quadratic function.

**USING STRUCTURE** **In Exercises 27–30, describe the transformation of the graph of the parent quadratic function. Then identify the vertex.**

27. $f(x) = 3(x + 2)^2 + 1$
28. $f(x) = -4(x + 1)^2 - 5$
29. $f(x) = -2x^2 + 5$
30. $f(x) = \frac{1}{2}(x - 1)^2$

**In Exercises 31–34, write a rule for $g$ described by the transformations of the graph of $f$. Then identify the vertex.** *(See Examples 3 and 4.)*

**31.** $f(x) = x^2$; vertical stretch by a factor of 4 and a reflection in the $x$-axis, followed by a translation 2 units up

**32.** $f(x) = x^2$; vertical shrink by a factor of $\frac{1}{3}$ and a reflection in the $y$-axis, followed by a translation 3 units right

**33.** $f(x) = 8x^2 - 6$; horizontal stretch by a factor of 2 and a translation 2 units up, followed by a reflection in the $y$-axis

**34.** $f(x) = (x + 6)^2 + 3$; horizontal shrink by a factor of $\frac{1}{2}$ and a translation 1 unit down, followed by a reflection in the $x$-axis

**USING TOOLS In Exercises 35–40, match the function with its graph. Explain your reasoning.**

**35.** $g(x) = 2(x - 1)^2 - 2$ **36.** $g(x) = \frac{1}{2}(x + 1)^2 - 2$

**37.** $g(x) = -2(x - 1)^2 + 2$

**38.** $g(x) = 2(x + 1)^2 + 2$ **39.** $g(x) = -2(x + 1)^2 - 2$

**40.** $g(x) = 2(x - 1)^2 + 2$

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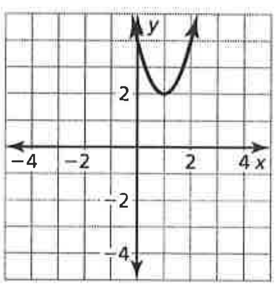


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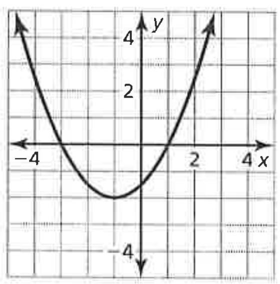


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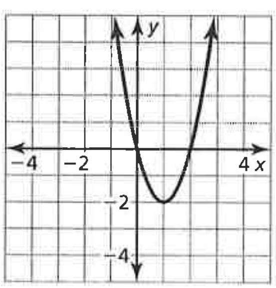


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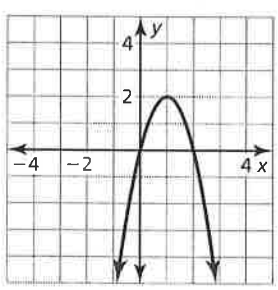


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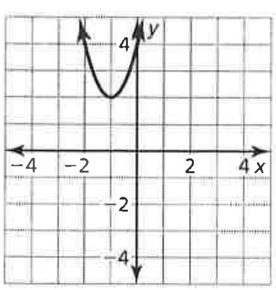


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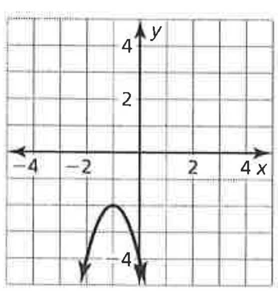



**JUSTIFYING STEPS In Exercises 41 and 42, justify each step in writing a function $g$ based on the transformations of $f(x) = 2x^2 + 6x$.**

**41.** translation 6 units down followed by a reflection in the $x$-axis

$h(x) = f(x) - 6$
$= 2x^2 + 6x - 6$
$g(x) = -h(x)$
$= -(2x^2 + 6x - 6)$
$= -2x^2 - 6x + 6$

**42.** reflection in the $y$-axis followed by a translation 4 units right

$h(x) = f(-x)$
$= 2(-x)^2 + 6(-x)$
$= 2x^2 - 6x$
$g(x) = h(x - 4)$
$= 2(x - 4)^2 - 6(x - 4)$
$= 2x^2 - 22x + 56$

**43. MODELING WITH MATHEMATICS** The function $h(x) = -0.03(x - 14)^2 + 6$ models the jump of a red kangaroo, where $x$ is the horizontal distance traveled (in feet) and $h(x)$ is the height (in feet). When the kangaroo jumps from a higher location, it lands 5 feet farther away. Write a function that models the second jump. *(See Example 5.)*

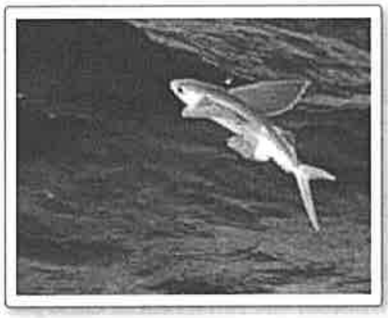
**44. MODELING WITH MATHEMATICS** The function $f(t) = -16t^2 + 10$ models the height (in feet) of an object $t$ seconds after it is dropped from a height of 10 feet on Earth. The same object dropped from the same height on the moon is modeled by $g(t) = -\frac{8}{3}t^2 + 10$. Describe the transformation of the graph of $f$ to obtain $g$. From what height must the object be dropped on the moon so it hits the ground at the same time as on Earth?

**45. MODELING WITH MATHEMATICS** Flying fish use their pectoral fins like airplane wings to glide through the air.

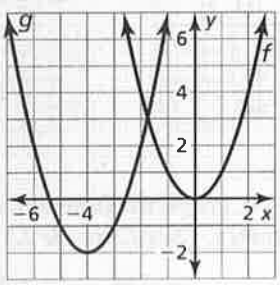
**a.** Write an equation of the form $y = a(x - h)^2 + k$ with vertex (33, 5) that models the flight path, assuming the fish leaves the water at (0, 0).

**b.** What are the domain and range of the function? What do they represent in this situation?

**c.** Does the value of $a$ change when the flight path has vertex (30, 4)? Justify your answer.



**46. HOW DO YOU SEE IT?** Describe the graph of $g$ as a transformation of the graph of $f(x) = x^2$.




**47. COMPARING METHODS** Let the graph of $g$ be a translation 3 units up and 1 unit right followed by a vertical stretch by a factor of 2 of the graph of $f(x) = x^2$.

**a.** Identify the values of $a$, $h$, and $k$ and use vertex form to write the transformed function.

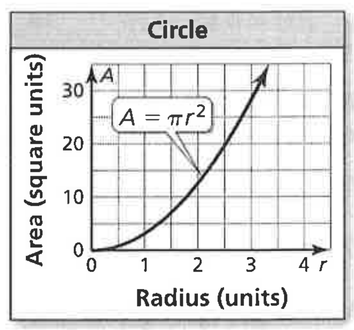
**b.** Use function notation to write the transformed function. Compare this function with your function in part (a).

**c.** Suppose the vertical stretch was performed first, followed by the translations. Repeat parts (a) and (b).

**d.** Which method do you prefer when writing a transformed function? Explain.

**48. THOUGHT PROVOKING** A jump on a pogo stick with a conventional spring can be modeled by $f(x) = -0.5(x - 6)^2 + 18$, where $x$ is the horizontal distance (in inches) and $f(x)$ is the vertical distance (in inches). Write at least one transformation of the function and provide a possible reason for your transformation.

**49. MATHEMATICAL CONNECTIONS** The area of a circle depends on the radius, as shown in the graph. A circular earring with a radius of $r$ millimeters has a circular hole with a radius of $\frac{3r}{4}$ millimeters. Describe a transformation of the graph below that models the area of the blue portion of the earring.

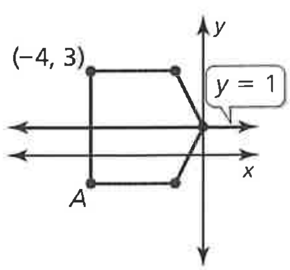



## Maintaining Mathematical Proficiency

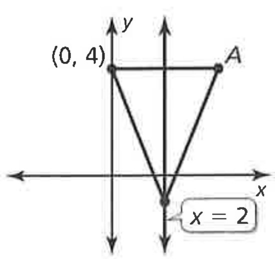
Reviewing what you learned in previous grades and lessons

**A line of symmetry for the figure is shown in red. Find the coordinates of point $A$.** *(Skills Review Handbook)*

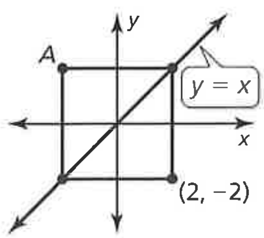
**50.**




**51.**




**52.**



# 2.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **WRITING** Explain how to determine whether a quadratic function will have a minimum value or a maximum value.

2. **WHICH ONE DOESN'T BELONG?** The graph of which function does *not* belong with the other three? Explain.

| | |
|---|---|
| $f(x) = 3x^2 + 6x - 24$ | $f(x) = 3x^2 + 24x - 6$ |
| $f(x) = 3(x - 2)(x + 4)$ | $f(x) = 3(x + 1)^2 - 27$ |

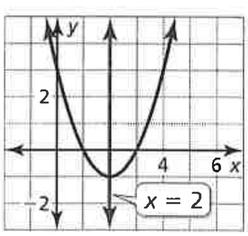
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–14, graph the function. Label the vertex and axis of symmetry.** *(See Example 1.)*

3. $f(x) = (x - 3)^2$
4. $h(x) = (x + 4)^2$
5. $g(x) = (x + 3)^2 + 5$
6. $y = (x - 7)^2 - 1$
7. $y = -4(x - 2)^2 + 4$
8. $g(x) = 2(x + 1)^2 - 3$
9. $f(x) = -2(x - 1)^2 - 5$
10. $h(x) = 4(x + 4)^2 + 6$
11. $y = -\frac{1}{4}(x + 2)^2 + 1$
12. $y = \frac{1}{2}(x - 3)^2 + 2$
13. $f(x) = 0.4(x - 1)^2$
14. $g(x) = 0.75x^2 - 5$

**ANALYZING RELATIONSHIPS In Exercises 15–18, use the axis of symmetry to match the equation with its graph.**

15. $y = 2(x - 3)^2 + 1$
16. $y = (x + 4)^2 - 2$
17. $y = \frac{1}{2}(x + 1)^2 + 3$
18. $y = (x - 2)^2 - 1$

A.
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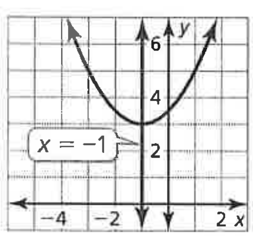


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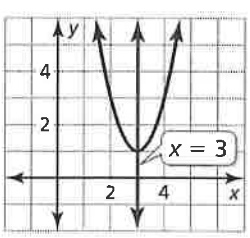


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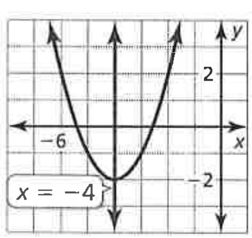


**REASONING In Exercises 19 and 20, use the axis of symmetry to plot the reflection of each point and complete the parabola.**

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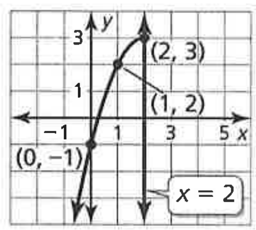


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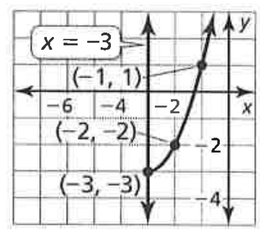



**In Exercises 21–30, graph the function. Label the vertex and axis of symmetry.** *(See Example 2.)*

21. $y = x^2 + 2x + 1$
22. $y = 3x^2 - 6x + 4$
23. $y = -4x^2 + 8x + 2$
24. $f(x) = -x^2 - 6x + 3$
25. $g(x) = -x^2 - 1$
26. $f(x) = 6x^2 - 5$
27. $g(x) = -1.5x^2 + 3x + 2$
28. $f(x) = 0.5x^2 + x - 3$
29. $y = \frac{3}{2}x^2 - 3x + 6$
30. $y = -\frac{5}{2}x^2 - 4x - 1$

31. **WRITING** Two quadratic functions have graphs with vertices (2, 4) and (2, −3). Explain why you can not use the axes of symmetry to distinguish between the two functions.

32. **WRITING** A quadratic function is increasing to the left of $x = 2$ and decreasing to the right of $x = 2$. Will the vertex be the highest or lowest point on the graph of the parabola? Explain.

**ERROR ANALYSIS** **In Exercises 33 and 34, describe and correct the error in analyzing the graph of $y = 4x^2 + 24x - 7$.**

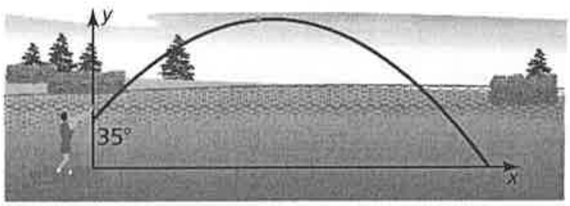
33. ✗ The x-coordinate of the vertex is $x = \frac{b}{2a} = \frac{24}{2(4)} = 3.$

34. ✗ The y-intercept of the graph is the value of c, which is 7.

**MODELING WITH MATHEMATICS** **In Exercises 35 and 36, $x$ is the horizontal distance (in feet) and $y$ is the vertical distance (in feet). Find and interpret the coordinates of the vertex.**

35. The path of a basketball thrown at an angle of 45° can be modeled by $y = -0.02x^2 + x + 6$.

36. The path of a shot put released at an angle of 35° can be modeled by $y = -0.01x^2 + 0.7x + 6$.




37. **ANALYZING EQUATIONS** The graph of which function has the same axis of symmetry as the graph of $y = x^2 + 2x + 2$?

Ⓐ $y = 2x^2 + 2x + 2$

Ⓑ $y = -3x^2 - 6x + 2$

Ⓒ $y = x^2 - 2x + 2$

Ⓓ $y = -5x^2 + 10x + 2$

38. **USING STRUCTURE** Which function represents the widest parabola? Explain your reasoning.

Ⓐ $y = 2(x + 3)^2$

Ⓑ $y = x^2 - 5$

Ⓒ $y = 0.5(x - 1)^2 + 1$

Ⓓ $y = -x^2 + 6$

**In Exercises 39–48, find the minimum or maximum value of the function. Describe the domain and range of the function, and where the function is increasing and decreasing.** *(See Example 3.)*

39. $y = 6x^2 - 1$

40. $y = 9x^2 + 7$

41. $y = -x^2 - 4x - 2$

42. $g(x) = -3x^2 - 6x + 5$

43. $f(x) = -2x^2 + 8x + 7$

44. $g(x) = 3x^2 + 18x - 5$

45. $h(x) = 2x^2 - 12x$

46. $h(x) = x^2 - 4x$

47. $y = \frac{1}{4}x^2 - 3x + 2$

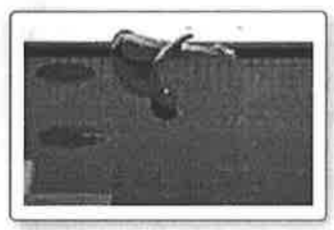
48. $f(x) = \frac{3}{2}x^2 + 6x + 4$

49. **PROBLEM SOLVING** The path of a diver is modeled by the function $f(x) = -9x^2 + 9x + 1$, where $f(x)$ is the height of the diver (in meters) above the water and $x$ is the horizontal distance (in meters) from the end of the diving board.

  a. What is the height of the diving board?

  b. What is the maximum height of the diver?

  c. Describe where the diver is ascending and where the diver is descending.



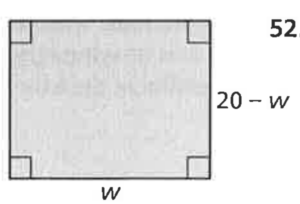
50. **PROBLEM SOLVING** The engine torque $y$ (in foot-pounds) of one model of car is given by $y = -3.75x^2 + 23.2x + 38.8$, where $x$ is the speed (in thousands of revolutions per minute) of the engine.

  a. Find the engine speed that maximizes torque. What is the maximum torque?

  b. Explain what happens to the engine torque as the speed of the engine increases.

**MATHEMATICAL CONNECTIONS** **In Exercises 51 and 52, write an equation for the area of the figure. Then determine the maximum possible area of the figure.**

51.

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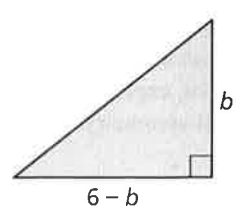


**In Exercises 53–60, graph the function. Label the $x$-intercept(s), vertex, and axis of symmetry.** *(See Example 4.)*

**53.** $y = (x + 3)(x - 3)$

**54.** $y = (x + 1)(x - 3)$

**55.** $y = 3(x + 2)(x + 6)$

**56.** $f(x) = 2(x - 5)(x - 1)$

**57.** $g(x) = -x(x + 6)$

**58.** $y = -4x(x + 7)$

**59.** $f(x) = -2(x - 3)^2$

**60.** $y = 4(x - 7)^2$

**USING TOOLS In Exercises 61–64, identify the $x$-intercepts of the function and describe where the graph is increasing and decreasing. Use a graphing calculator to verify your answer.**

**61.** $f(x) = \frac{1}{2}(x - 2)(x + 6)$

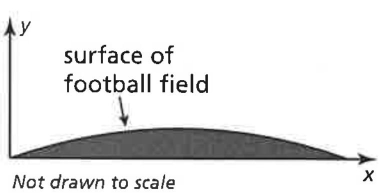
**62.** $y = \frac{3}{4}(x + 1)(x - 3)$

**63.** $g(x) = -4(x - 4)(x - 2)$

**64.** $h(x) = -5(x + 5)(x + 1)$

**65. MODELING WITH MATHEMATICS** A soccer player kicks a ball downfield. The height of the ball increases until it reaches a maximum height of 8 yards, 20 yards away from the player. A second kick is modeled by $y = x(0.4 - 0.008x)$. Which kick travels farther before hitting the ground? Which kick travels higher? *(See Example 5.)*

**66. MODELING WITH MATHEMATICS** Although a football field appears to be flat, some are actually shaped like a parabola so that rain runs off to both sides. The cross section of a field can be modeled by $y = -0.000234x(x - 160)$, where $x$ and $y$ are measured in feet. What is the width of the field? What is the maximum height of the surface of the field?




**67. REASONING** The points $(2, 3)$ and $(-4, 2)$ lie on the graph of a quadratic function. Determine whether you can use these points to find the axis of symmetry. If not, explain. If so, write the equation of the axis of symmetry.

**68. OPEN-ENDED** Write two different quadratic functions in intercept form whose graphs have the axis of symmetry $x = 3$.

**69. PROBLEM SOLVING** An online music store sells about 4000 songs each day when it charges \$1 per song. For each \$0.05 increase in price, about 80 fewer songs per day are sold. Use the verbal model and quadratic function to determine how much the store should charge per song to maximize daily revenue.

Revenue (dollars) = Price (dollars/song) • Sales (songs)

$R(x) = (1 + 0.05x) \cdot (4000 - 80x)$

**70. PROBLEM SOLVING** An electronics store sells 70 digital cameras per month at a price of \$320 each. For each \$20 decrease in price, about 5 more cameras per month are sold. Use the verbal model and quadratic function to determine how much the store should charge per camera to maximize monthly revenue.

Revenue (dollars) = Price (dollars/camera) • Sales (cameras)

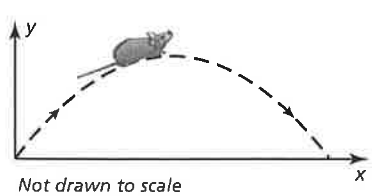
$R(x) = (320 - 20x) \cdot (70 + 5x)$

**71. DRAWING CONCLUSIONS** Compare the graphs of the three quadratic functions. What do you notice? Rewrite the functions $f$ and $g$ in standard form to justify your answer.

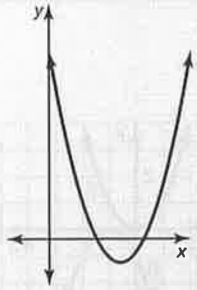
$f(x) = (x + 3)(x + 1)$
$g(x) = (x + 2)^2 - 1$
$h(x) = x^2 + 4x + 3$

**72. USING STRUCTURE** Write the quadratic function $f(x) = x^2 + x - 12$ in intercept form. Graph the function. Label the $x$-intercepts, $y$-intercept, vertex, and axis of symmetry.

**73. PROBLEM SOLVING** A woodland jumping mouse hops along a parabolic path given by $y = -0.2x^2 + 1.3x$, where $x$ is the mouse's horizontal distance traveled (in feet) and $y$ is the corresponding height (in feet). Can the mouse jump over a fence that is 3 feet high? Justify your answer.



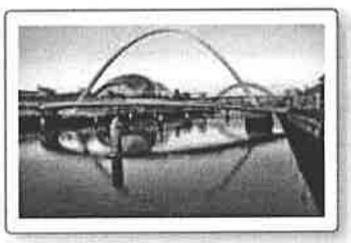
74. **HOW DO YOU SEE IT?** Consider the graph of the function $f(x) = a(x - p)(x - q)$.




a. What does $f\left(\frac{p + q}{2}\right)$ represent in the graph?

b. If $a < 0$, how does your answer in part (a) change? Explain.

75. **MODELING WITH MATHEMATICS** The Gateshead Millennium Bridge spans the River Tyne. The arch of the bridge can be modeled by a parabola. The arch reaches a maximum height of 50 meters at a point roughly 63 meters across the river. Graph the curve of the arch. What are the domain and range? What do they represent in this situation?



76. **THOUGHT PROVOKING** You have 100 feet of fencing to enclose a rectangular garden. Draw three possible designs for the garden. Of these, which has the greatest area? Make a conjecture about the dimensions of the rectangular garden with the greatest possible area. Explain your reasoning.

77. **MAKING AN ARGUMENT** The point (1, 5) lies on the graph of a quadratic function with axis of symmetry $x = -1$. Your friend says the vertex could be the point (0, 5). Is your friend correct? Explain.

78. **CRITICAL THINKING** Find the $y$-intercept in terms of $a$, $p$, and $q$ for the quadratic function $f(x) = a(x - p)(x - q)$.

79. **MODELING WITH MATHEMATICS** A kernel of popcorn contains water that expands when the kernel is heated, causing it to pop. The equations below represent the "popping volume" $y$ (in cubic centimeters per gram) of popcorn with moisture content $x$ (as a percent of the popcorn's weight).

**Hot-air popping:** $y = -0.761(x - 5.52)(x - 22.6)$

**Hot-oil popping:** $y = -0.652(x - 5.35)(x - 21.8)$

a. For hot-air popping, what moisture content maximizes popping volume? What is the maximum volume?

b. For hot-oil popping, what moisture content maximizes popping volume? What is the maximum volume?

c. Use a graphing calculator to graph both functions in the same coordinate plane. What are the domain and range of each function in this situation? Explain.

80. **ABSTRACT REASONING** A function is written in intercept form with $a > 0$. What happens to the vertex of the graph as $a$ increases? as $a$ approaches 0?

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation. Check for extraneous solutions.** *(Skills Review Handbook)*

81. $3\sqrt{x} - 6 = 0$

82. $2\sqrt{x - 4} - 2 = 2$

83. $\sqrt{5x} + 5 = 0$

84. $\sqrt{3x + 8} = \sqrt{x + 4}$

**Solve the proportion.** *(Skills Review Handbook)*

85. $\frac{1}{2} = \frac{x}{4}$

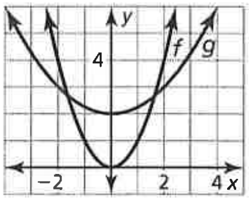
86. $\frac{2}{3} = \frac{x}{9}$

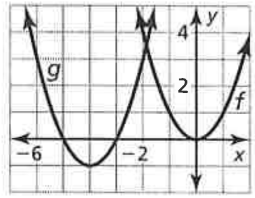
87. $\frac{-1}{4} = \frac{3}{x}$

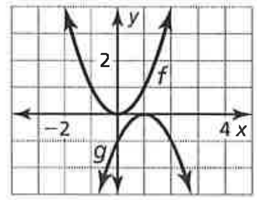
88. $\frac{5}{2} = \frac{-20}{x}$

# 2.1–2.2 Quiz

**Describe the transformation of $f(x) = x^2$ represented by $g$.** ***(Section 2.1)***

1. 


2. 


3. 


**Write a rule for $g$ and identify the vertex.** ***(Section 2.1)***

4. Let $g$ be a translation 2 units up followed by a reflection in the $x$-axis and a vertical stretch by a factor of 6 of the graph of $f(x) = x^2$.

5. Let $g$ be a translation 1 unit left and 6 units down, followed by a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = 3(x + 2)^2$.

6. Let $g$ be a horizontal shrink by a factor of $\frac{1}{4}$, followed by a translation 1 unit up and 3 units right of the graph of $f(x) = (2x + 1)^2 - 11$.

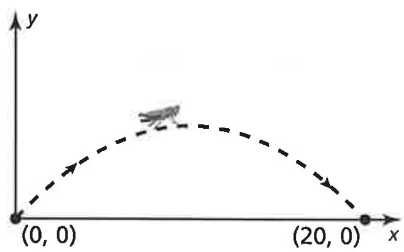
**Graph the function. Label the vertex and axis of symmetry.** ***(Section 2.2)***

7. $f(x) = 2(x - 1)^2 - 5$
8. $h(x) = 3x^2 + 6x - 2$
9. $f(x) = 7 - 8x - x^2$

**Find the $x$-intercepts of the graph of the function. Then describe where the function is increasing and decreasing.** ***(Section 2.2)***

10. $g(x) = -3(x + 2)(x + 4)$
11. $g(x) = \frac{1}{2}(x - 5)(x + 1)$
12. $f(x) = 0.4x(x - 6)$

13. A grasshopper can jump incredible distances, up to 20 times its length. The height (in inches) of the jump above the ground of a 1-inch-long grasshopper is given by $h(x) = -\frac{1}{20}x^2 + x$, where $x$ is the horizontal distance (in inches) of the jump. When the grasshopper jumps off a rock, it lands on the ground 2 inches farther. Write a function that models the new path of the jump. *(Section 2.1)*




*Not drawn to scale*

14. A passenger on a stranded lifeboat shoots a distress flare into the air. The height (in feet) of the flare above the water is given by $f(t) = -16t(t - 8)$, where $t$ is time (in seconds) since the flare was shot. The passenger shoots a second flare, whose path is modeled in the graph. Which flare travels higher? Which remains in the air longer? Justify your answer. *(Section 2.2)*