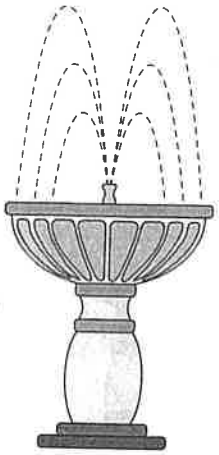
THE COIN FOUNTAIN

The study of parabolic curves through the design of water arcs



You have been hired to design the water arc of a coin fountain. The pool of the fountain is 20 feet wide, and the water arc is to be greater than 6 feet tall, but less than 50 feet. You will need to determine the locations of the launch point and landing points and the maximum height of the arc. Also, you will need to write an equation that describes the water arc in terms of its height in relation to the horizontal distance along the pool.

PART ONE

1. Place the side view of your fountain in a first quadrant graph. Have the surface of the pool correspond to the x-axis with the left side at the origin. Show the coordinates of the roots and vertex, and include all pertinent data points from the following questions.

2. Using the equation $y = a(x - x_1)(x - x_2)$ where $(x_1, 0)$ & $(x_2, 0)$ are the roots of the parabola, choose a value for "a" that will produce a reasonable arc. Then convert your equation to the form: $y = ax^2 + bx + c$.

3. State your launch and landing points and the height of your water arc.

   Launch (,)
   Landing (,)
   Height _____

4. After one foot of horizontal distance (from your launch point) how high will the water arc be?

5. Find the vertex and the equation for the axis of symmetry.

   Vertex (,)
   Axis: x =

THE COIN FOUNTAIN

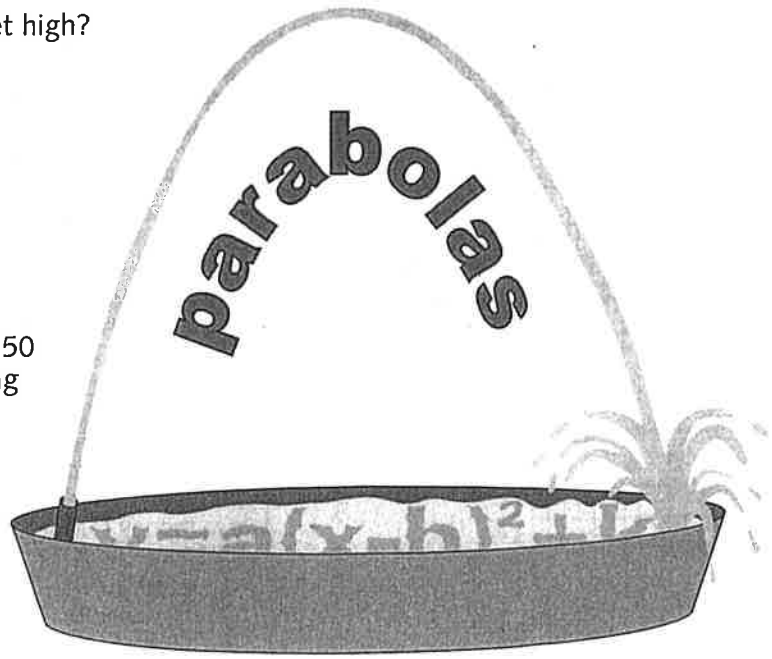
PART TWO

6. Factor your equation (in the form $y = ax^2 + bx + c$) to find the roots of the equation and then verify your launch and landing points.

7. Use the quadratic formula to find the roots, again verifying the launch and landing points.

8. Convert your original equation from quadratic form to vertex form, $y= a(x - h)^2 + k$, by completing the square.

9. At what horizontal distance will the arc be 5 feet high?



PART THREE

10. Determine the value of "a" that will produce a 50 foot high arc for your chosen launch and landing points.

11. Create a second arc that intersects the first one and find their point of intersection.

12. Create and graph the coordinates of the focus and the equation for the directrix of your water arc.