

Simplify.

1.  $\sqrt{16} = 4$   
4 4

2.  $\sqrt{64} = 8$   
8 8

3.  $\sqrt{225} = 15$   
15 15

4.  $\sqrt{2025} = 45$   
45 45

5.  $\sqrt{57,600} = 240$   
240 240

6.  $\sqrt{36} = 6$   
6 6

7.  $\sqrt{400}$   
20 20

8.  $\sqrt{4}$   
2 2

9.  $\sqrt{3600} = 60$   
60 60

Square root: need to find two factors that are the same number  $\rightarrow$  that when multiplied together equal the number under the radical.

Warm Up

Determine whether the function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

1.  $y = 5(0.7)^t$   
growth

2.  $y = 49(1.04)^t$   
growth

3.  $r(t) = 0.5(0.95)^t$   
decay

4.  $g(t) = 3\left(\frac{4}{5}\right)^t$  decay

Cumulative Warm Up

## Essential Question

How can you multiply and divide square roots?

- are they the same as working with variables.

What you will learn:

- Use properties of radicals to simplify expressions
- Simplify expressions by rationalizing the denominator
- Perform operations with radicals

Essential Question

### Core Concept

#### Product Property of Square Roots

**Words** The square root of a product equals the product of the square roots of the factors.

**Numbers**  $\sqrt{9} \cdot \sqrt{5} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

**Algebra**  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ , where  $a, b \geq 0$

Radical Expression: an expression that contains a radical

Simplest Form:

- No radicands have perfect  $n^{\text{th}}$  powers a factors other than 1.
- No radicand contains a fraction
- No radicals appear in the denominator of a fraction

Core Concept

a.  $\sqrt{108} = \sqrt{36 \cdot 3}$  Factor using the greatest perfect square factor.  
 $= \sqrt{36} \cdot \sqrt{3}$  Product Property of Square Roots  
 $= 6\sqrt{3}$  Simplify.

b.  $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$  Factor using the greatest perfect square factor.  
 $= \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x}$  Product Property of Square Roots  
 $= 3x\sqrt{x}$  Simplify.

• Use factor trees and factor to prime numbers

• Always look for pairs when working w/ square root

Index  $\rightarrow \sqrt{\quad}$

Index tells us to look for pairs

• leave any value that does not have a pair under the radical

Example 1

Simplify the expression.

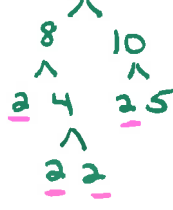
1.  $\sqrt{24}$



$2\sqrt{2 \cdot 3}$

$2\sqrt{6}$

2.  $-\sqrt{80}$



$-1 \cdot 2 \cdot 2 \sqrt{5}$

$-4\sqrt{5}$

3.  $\sqrt{49x^3}$



$7 \cdot x \sqrt{x}$

$7x\sqrt{x}$

4.  $\sqrt{75n^5}$



$5 \cdot n \cdot n \sqrt{3 \cdot n}$

$5n^2\sqrt{3n}$

### Core Concept

#### Quotient Property of Square Roots

**Words** The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

**Numbers**  $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$       **Algebra**  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ , where  $a \geq 0$  and  $b > 0$

• If one big radical - you can do the square of the numerator and the square of the denominator independently.

Core Concept 2

a.  $\sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{\sqrt{64}}$       Quotient Property of Square Roots  
 $= \frac{\sqrt{15}}{8}$       Simplify.

b.  $\sqrt{\frac{81}{x^2}} = \frac{\sqrt{81}}{\sqrt{x^2}}$       Quotient Property of Square Roots  
 $= \frac{9}{x}$       Simplify.

Example 2

\* Index number tells us how many grouping we need!

a.  $\sqrt[3]{-128} = \sqrt[3]{-64 \cdot 2}$  Factor using the greatest perfect cube factor.  
 $= \sqrt[3]{-64} \cdot \sqrt[3]{2}$  Product Property of Cube Roots  
 $= -4\sqrt[3]{2}$  Simplify.

b.  $\sqrt[3]{125x^7} = \sqrt[3]{125 \cdot x^6 \cdot x}$  Factor using the greatest perfect cube factors.  
 $= \sqrt[3]{125} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{x}$  Product Property of Cube Roots  
 $= 5x^2\sqrt[3]{x}$  Simplify.

c.  $\sqrt[3]{\frac{y}{216}} = \frac{\sqrt[3]{y}}{\sqrt[3]{216}}$  Quotient Property of Cube Roots  
 $= \frac{\sqrt[3]{y}}{6}$  Simplify.

d.  $\sqrt[3]{\frac{8x^4}{27y^3}} = \frac{\sqrt[3]{8x^4}}{\sqrt[3]{27y^3}}$  Quotient Property of Cube Roots  
 $= \frac{\sqrt[3]{8 \cdot x^3 \cdot x}}{\sqrt[3]{27 \cdot y^3}}$  Factor using the greatest perfect cube factors.  
 $= \frac{\sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}}{\sqrt[3]{27} \cdot \sqrt[3]{y^3}}$  Product Property of Cube Roots  
 $= \frac{2x\sqrt[3]{x}}{3y}$  Simplify.

Example 3

Simplify the expression.

5.  $\sqrt{\frac{23}{9}}$   $\frac{\sqrt{23}}{\sqrt{9}} = \frac{\sqrt{23}}{3}$

6.  $-\sqrt{\frac{17}{100}}$   $-\frac{\sqrt{17}}{\sqrt{100}} = -\frac{\sqrt{17}}{10}$

7.  $\sqrt{\frac{36}{z^2}} = \frac{\sqrt{36}}{\sqrt{z^2}} = \frac{6}{z}$

8.  $\sqrt{\frac{4x^2}{64}}$   $\frac{\sqrt{4x^2}}{\sqrt{64}} = \frac{2x}{8} = \frac{1}{4}x$

9.  $\sqrt[3]{54}$   
 $\begin{matrix} 6 & \wedge & 9 \\ \wedge & & \wedge \\ 2 & 3 & 3 \end{matrix}$   
 Index = 3

10.  $\sqrt[3]{16x^4}$   
 $\begin{matrix} 4 & \wedge & 4 \\ \wedge & & \wedge \\ 2 & 2 & 2 \end{matrix}$   
 $2x\sqrt[3]{2x}$

11.  $\sqrt[3]{\frac{a}{-27}}$   $\frac{\sqrt[3]{a}}{\sqrt[3]{-27}} = \frac{\sqrt[3]{a}}{-3}$

12.  $\sqrt[3]{\frac{25c^7d^3}{64}}$   $\frac{\sqrt[3]{25c^7d^3}}{\sqrt[3]{64}}$   
 $\frac{5 \cdot 5 \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c}{4 \cdot 4 \cdot 4}$   
 $\frac{c^2 d \sqrt[3]{25c}}{4}$

• Can not have a radical in the denominator

• Remove a radical by rationalizing the denominator

<p>a. <math>\frac{\sqrt{5}}{\sqrt{3n}} = \frac{\sqrt{5}}{\sqrt{3n}} \cdot \frac{\sqrt{3n}}{\sqrt{3n}}</math></p> <p><math>= \frac{\sqrt{15n}}{\sqrt{9n^2}}</math></p> <p><math>= \frac{\sqrt{15n}}{\sqrt{9} \cdot \sqrt{n^2}}</math></p> <p><math>= \frac{\sqrt{15n}}{3n}</math></p>	<p>Multiply by <math>\frac{\sqrt{3n}}{\sqrt{3n}}</math>.</p> <p>Product Property of Square Roots</p> <p>Product Property of Square Roots</p> <p>Simplify.</p>
<p>b. <math>\frac{2}{\sqrt[3]{9}} = \frac{2}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}</math></p> <p><math>= \frac{2\sqrt[3]{3}}{\sqrt[3]{27}}</math></p> <p><math>= \frac{2\sqrt[3]{3}}{3}</math></p>	<p>Multiply by <math>\frac{\sqrt[3]{3}}{\sqrt[3]{3}}</math>.</p> <p>Product Property of Cube Roots</p> <p>Simplify.</p>

Example 4

	2	-√3
2	4	-2√3
+√3	2√3	-√9
		-3

4 - 3 = 1

Simplify  $\frac{7}{2-\sqrt{3}} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$\frac{(7 \cdot 2 + 7 \cdot \sqrt{3})}{1} = \boxed{14 + 7\sqrt{3}}$

Example 5

Simplify the expression.

13.  $\frac{1}{\sqrt{5}}$   $\frac{(\sqrt{5})}{(\sqrt{5})}$

$$\frac{\sqrt{5}}{5}$$

14.  $\frac{\sqrt{10}}{\sqrt{3}}$   $\frac{(\sqrt{3})}{(\sqrt{3})}$

$$\frac{\sqrt{30}}{3}$$

15.  $\frac{7}{\sqrt{2x}}$   $\frac{(\sqrt{2x})}{(\sqrt{2x})}$

$$\frac{7\sqrt{2x}}{2x}$$

16.  $\sqrt{\frac{2y^2}{3}}$

$$\frac{\sqrt{2}y}{\sqrt{3}}$$

$$\frac{4\sqrt{6}}{3}$$

17.  $\frac{5}{\sqrt[3]{32}}$

$$\frac{5}{2\sqrt[3]{4}}$$

$$\frac{5\sqrt[3]{2}}{4}$$

18.  $\frac{8}{1+\sqrt{3}}$

$$\frac{8(1-\sqrt{3})}{1+\sqrt{3}(1-\sqrt{3})}$$

$$\frac{-4 + 4\sqrt{3}}{+2}$$

$$-4 + 4\sqrt{3}$$

19.  $\frac{\sqrt{13}}{\sqrt{5}-2}$

20.  $\frac{12}{\sqrt{2}+\sqrt{7}}$

Handwritten notes on the left side of the page showing a sequence of numbers: 2, 3, 2, 1, 4, 2, 8, a, 4, 2, 2. Some numbers are underlined or have arrows pointing to them.

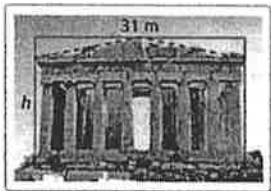
Monitoring Progress 13-20

The distance  $d$  (in miles) that you can see to the horizon with your eye level  $h$  feet above the water is given by  $d = \sqrt{\frac{3h}{2}}$ . How far can you see when your eye level is 5 feet above the water?



Example 6

The ratio of the length to the width of a *golden rectangle* is  $(1+\sqrt{5}) : 2$ . The dimensions of the face of the Parthenon in Greece form a golden rectangle. What is the height  $h$  of the Parthenon?



Example 7

21. WHAT IF? In Example 6, how far can you see when your eye level is 35 feet above the water?

22. The dimensions of a dance floor form a golden rectangle. The shorter side of the dance floor is 50 feet. What is the length of the longer side of the dance floor?



• Like terms

• If values

under the radical  
are the same  
you can add or  
subtract the  
coefficients

$$\begin{aligned} \text{a. } 5\sqrt{7} + \sqrt{11} - 8\sqrt{7} &= 5\sqrt{7} - 8\sqrt{7} + \sqrt{11} && \text{Commutative Property of Addition} \\ &= (5-8)\sqrt{7} + \sqrt{11} && \text{Distributive Property} \\ &= -3\sqrt{7} + \sqrt{11} && \text{Subtract.} \end{aligned}$$

$$\begin{aligned} \text{b. } 10\sqrt{5} + \sqrt{20} &= 10\sqrt{5} + \sqrt{4 \cdot 5} && \text{Factor using the greatest perfect} \\ & && \text{square factor.} \\ &= 10\sqrt{5} + \sqrt{4} \cdot \sqrt{5} && \text{Product Property of Square Roots} \\ &= 10\sqrt{5} + 2\sqrt{5} && \text{Simplify.} \\ &= (10+2)\sqrt{5} && \text{Distributive Property} \\ &= 12\sqrt{5} && \text{Add.} \end{aligned}$$

$$\begin{aligned} \text{c. } 6\sqrt[3]{x} + 2\sqrt[3]{x} &= (6+2)\sqrt[3]{x} && \text{Distributive Property} \\ &= 8\sqrt[3]{x} && \text{Add.} \end{aligned}$$

Example 8

distributive  
property

Simplify  $\sqrt{5}(\sqrt{3} - \sqrt{75})$ .

$$(\sqrt{5} \cdot \sqrt{3}) + (\sqrt{5} \cdot -\sqrt{75}) \times \frac{5}{15}$$

$$\sqrt{15} + -5\sqrt{15}$$

$$-4\sqrt{15}$$

Example 9

Simplify the expression.

23.  $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$

$$13\sqrt{2} - \sqrt{6}$$

24.  $4\sqrt{7} - 6\sqrt{63}$   $\leftarrow \begin{matrix} 9 \\ 7 \end{matrix}$

$$4\sqrt{7} - 18\sqrt{7}$$

$$-14\sqrt{7}$$

25.  $4\sqrt[3]{5x} - 11\sqrt[3]{5x}$

$$-7\sqrt[3]{5x}$$

26.  $\sqrt{3}(8\sqrt{2} + 7\sqrt{32})$

$$8\sqrt{6} + 7\sqrt{96}$$
  $\leftarrow \begin{matrix} 6 \\ 16 \end{matrix}$

$$8\sqrt{6} + 7 \cdot 4\sqrt{6}$$

$$8 + 28\sqrt{6}$$

$$36\sqrt{6}$$

27.  $(2\sqrt{5} - 4)^2$

$$(2\sqrt{5} - 4)(2\sqrt{5} - 4)$$

28.  $\sqrt[3]{-4}(\sqrt[3]{2} - \sqrt[3]{16})$

Monitoring Progress 23-28

**Writing Prompt:** This was a very long lesson. Right now I am feeling ...

Closure