Find the sum.

1. $\sum_{n=1}^{\text{[illegible]}} 2^{n-1}$
2. $\sum_{n=1}^{\text{[illegible]}} 2n-1$
3. $\sum_{n=1}^{\text{[illegible]}} 4\left(\frac{2}{3}\right)^{n-1}$
4. $\sum_{n=1}^{12} -2(3)^{n-1}$
5. $\sum_{n=1}^{200} 5-\frac{1}{2}n$
6. $\sum_{n=1}^{15} \frac{1}{2}(3)^{n-1}$

Warm Up

Graph the function. State the domain and range.

1. $f(x) = \frac{1}{x-1}+3$
2. $f(x) = \frac{-2}{x+3}$
3. $f(x) = \frac{3}{x-2}+4$
4. $f(x) = \frac{x}{x+5}$
5. $f(x) = \frac{3x-1}{2x-1}$
6. $f(x) = \frac{-2x+5}{x-3}$

Cumulative Warm Up

Essential Question

How can you find the sum of an infinite geometric series?

Essential Question

Work with a partner. Enter each geometric series in a spreadsheet. Then use the spreadsheet to determine whether the infinite geometric series has a finite sum. If it does, find the sum. Explain your reasoning. (The figure shows a partially completed spreadsheet for part (a).)

	A	B
1	1	1
2	2	0.5
3	3	0.25
4	4	0.125
5	5	0.0625
6	6	0.03125
7	7	
8	8	
9	9	
10	10	
11	11	
12	12	
13	13	
14	14	
15	15	
16	Sum	

a. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots$

b. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\dots$

c. $1+\frac{3}{2}+\frac{9}{4}+\frac{27}{8}+\frac{81}{16}+\dots$

Exploration 1 a-c

d. $1+\frac{5}{4}+\frac{25}{16}+\frac{125}{64}+\frac{625}{256}+\dots$

e. $1+\frac{4}{5}+\frac{16}{25}+\frac{64}{125}+\frac{256}{625}+\dots$

f. $1+\frac{9}{10}+\frac{81}{100}+\frac{729}{1000}+\frac{6561}{10,000}+\dots$

Exploration 1 d-f

Work with a partner. Look back at the infinite geometric series in Exploration 1. Write a conjecture about how you can determine whether the infinite geometric series

$$a_1 + a_1r + a_1r^2 + a_1r^3 + \dots$$

has a finite sum

Exploration 2

Work with a partner. In Lesson 8.3, you learned that the sum of the first n terms of a geometric series with first term a_1 and common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right).$$

When an infinite geometric series has a finite sum, what happens to r^n as n increases? Explain your reasoning. Write a formula to find the sum of an infinite geometric series. Then verify your formula by checking the sums you obtained in Exploration 1.

Exploration 3

Consider the infinite geometric series

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots$$

Find and graph the partial sums S_n for n = 1, 2, 3, 4, and 5. Then describe what happens to S_n as n increases.

Example 1

Core Concept

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1-r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

Core Concept

Find the sum of each infinite geometric series.

a. $\sum_{i=1}^{\infty} 3(0.7)^{i-1}$

b. $1 + 3 + 9 + 27 + \dots$

c. $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \dots$

Example 2

1. Consider the infinite geometric series

$$\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{1625} + \frac{32}{3125} + \dots$$

Find and graph the partial sums S_n for $n = 1, 2, 3, 4$, and 5. Then describe what happens to S_n as n increases.

Find the sum of the infinite geometric series, if it exists.

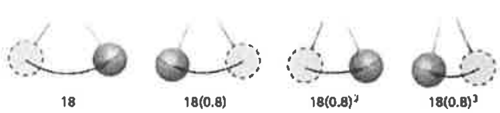
2. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

3. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1}$

4. $3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \dots$

Monitoring Progress 1-4

A pendulum that is released to swing freely travels 18 inches on the first swing. On each successive swing, the pendulum travels 80% of the distance of the previous swing. What is the total distance the pendulum swings?



Example 3

Write 0.242424 . . . as a fraction in simplest form.

Example 4

5. WHAT IF? In Example 3, suppose the pendulum travels 10 inches on its first swing. What is the total distance the pendulum swings?

Write the repeating decimal as a fraction in simplest form.

6. 0.555 . . . **7.** 0.727272 . . . **8.** 0.131313 . . .

Monitoring Progress 5-8

Exit Ticket: Determine the sum of the distances traveled by person B in the *Motivate* activity.

Closure