

Section 3.4 Notes

BL A2

Evaluate the expression when $a = -1$, $b = 3$, $c = 0$, and $d = 2$.

$$1. \frac{b-c}{d}$$

$$2. \frac{b-3d}{b}$$

$$3. b - d + 5a$$

$$4. 2(b + a) + bc$$

$$5. 2d + \frac{4}{9}b^3$$

$$6. -\frac{2}{5}b - 0(ac - bd)$$

Warm Up

Substitute given values
follow order of operations

Write a function g whose graph represents the indicated transformation of the graph of f . Use a graphing calculator to check your answer.

$$1. f(x) = -6x - 3; \text{ reflection in the } y\text{-axis}$$

$$2. f(x) = \frac{1}{3}x + 5; \text{ reflection in the } y\text{-axis}$$

Cumulative Warm Up

Essential Question

How can you derive a general formula for solving a quadratic equation?

Skip

Essential Question

What you will learn:

- Solve quadratics using the quadratic formula
- Analyze the discriminant to determine the number and types of solutions
- Solve real-life problems

Work with a partner. Analyze and describe what is done in each step in the development of the Quadratic Formula.

Step	Justification
$ax^2 + bx + c = 0$	
$ax^2 + bx = -c$	
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	
$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$	
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	
The result is the Quadratic Formula.	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Show steps as to how the quadratic formula is derived

Explain using the justification for each step.

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Identify each of the parts of a Standard form quadratic

Always set the quadratic equal to 0 before using quadratic formula.

Work with a partner. Use the Quadratic Formula to solve each equation.

a. $x^2 - 4x + 3 = 0$

b. $x^2 - 2x + 2 = 0$

c. $x^2 + 2x - 3 = 0$

d. $x^2 + 4x + 4 = 0$

e. $x^2 - 6x + 10 = 0$

f. $x^2 + 4x + 6 = 0$

Exploration 2

Core Concept

The Quadratic Formula

Let a , b , and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Core Concept

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Solve $x^2 + 3x - 5 = 0$ using the Quadratic Formula.

$$x^2 + 3x - 5 = 0$$
$$a = 1 \quad b = 3 \quad c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{9 + 20}}{2}$$
$$= \frac{-3 \pm \sqrt{29}}{2}$$

Example 1

Solve the equation using the Quadratic Formula.

$$1. x^2 - 6x + 4 = 0 \quad 2. 2x^2 + 4 = -7x \quad 3. 5x^2 = x + 8$$

Monitoring Progress 1-3

Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

$$-12x - 12x$$

$$25x^2 - 20x + 4 = 0$$

$$a = 25 \quad b = -20 \quad c = 4$$

Solution: $\frac{2}{5}$

Example 2

- Write In Standard form BI A2
- Define each part
- Substitute values into Quadratic Formula
- Use proper order of operations to solve..

Student practice

Always make sure that the original problem is in standard form.

Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

$$-x^2 + 4x - 13 = 0$$

$$a = -1 \quad b = 4 \quad c = -13$$

Solution:

$$x = 2 + 3i$$

$$x = 2 - 3i$$

Example 3

Solve the equation using the Quadratic Formula.

$$4. x^2 + 41 = -8x \quad 5. -9x^2 = 30x + 25 \quad 6. 5x - 7x^2 = 3x + 4$$

Monitoring Progress 4-6

Student practice

Core Concept

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$			

Two x-intercepts One x-intercept No x-intercept

Core Concept

discriminant is the portion of the quadratic formula under the radical symbol.

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$ b. $x^2 - 6x + 9 = 0$ c. $x^2 - 6x + 8 = 0$

$$\begin{aligned} (-6)^2 - 4(1)(10) &= \\ -4 \rightarrow \text{two } &\text{Imaginary} \\ (-6)^2 - 4(1)(9) &= \\ 0 \rightarrow \text{one real} \\ (-6)^2 - 4(1)(8) &= \\ 4 \rightarrow \text{two } &\text{real} \end{aligned}$$

Example 4

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

7. $4x^2 + 8x + 4 = 0$ 8. $\frac{1}{2}x^2 + x - 1 = 0$

9. $5x^2 = 8x - 13$

10. $7x^2 - 3x = 6$

11. $4x^2 + 6x = -9$

12. $-5x^2 + 1 = 6 - 10x$

Monitoring Progress 7-12

Find a possible pair of integer values for a and c so that the equation $ax^2 - 4x + c = 0$ has one real solution. Then write the equation.

$$\begin{aligned} b^2 - 4ac & \\ (-4)^2 - 4ac &= 0 \\ 16 - 4ac &= 0 \\ -16 & \quad -16 \\ -4ac &= -16 \\ \hline -4 & \quad -4 \\ ac &= 4 \end{aligned}$$

Example 5

discriminant

$b^2 - 4ac$

use the chart from
previous slide to determine
solutions

Student practice

because the solution
is $ac = 4$

choose two integers
whose product is 4

example: $a = 1$ $c = 4$

possible equation:

13. Find a possible pair of integer values for a and c so that the equation $ax^2 + 3x + c = 0$ has two real solutions. Then write the equation.

Monitoring Progress 13

Concept Summary

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for any quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for any quadratic equation.

Concept Summary

A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

$$\begin{aligned} h &= -16t^2 + V_0 t + h_0 \\ 3 &= -16t^2 + 30t + 4 \\ 0 &= -16t^2 + 30t + 1 \end{aligned}$$

Example 6

Student practice

review of different methods for solving quadratics

because the ball is thrown, use the given formula.

- Solve by using quadratic formula

- eliminate any solution that does not make sense.

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14. WHAT IF? The ball leaves the juggler's hand with an initial vertical velocity of 40 feet per second. How long is the ball in the air?

Monitoring Progress 14

Exit Ticket:

a. Give an example of a quadratic equation that you would *not* solve using the Quadratic Formula. Solve it.

b. Give an example of a quadratic equation that you would solve using the Quadratic Formula. Solve it.

Closure

