Simplify.

1. $4(x - 1) + 6(x + 6)$

   $10x + 32$

2. $7(y + 8) + (2 + 3y)$

   $10y + 58$

3. $3[x + 3(x + 2)]$

   $12x + 18$

4. $-4[x - 4(4 + x)]$

   $12x + 64$

5. $3x + 2[x + (5 + x)]$

   $7x + 10$

6. $-6x + 3[x + 5(x - 6)] + 8$

   $12x - 82$

Warm Up

* use algebraic properties to solve.
* distributive property
* combine like terms
* watch negatives

* check solutions against your work - ask questions if you don't understand

~~**Identify the vertex, focus, directrix, and axis of symmetry of the parabola.**~~

~~1. $y = \frac{1}{7}(x + 4)^2 - 1$~~

~~2. $y = \frac{1}{15}(x + 4)^2$~~

~~3. $y = -\frac{1}{8}(x - 3)^2$~~

~~4. $y = \frac{1}{4}(x - 4)^2 + 4$~~

Cumulative Warm Up

Skip

Essential Question

What are the subsets of the set of complex numbers?

* Imaginary unit only applies to even roots.

Essential Question

What you will learn:

* Define and use Imaginary unit i
* Add, subtract, and multiply complex numbers
* find complex solutions and zeros.

Work with a partner. Determine which subsets of the set of complex numbers contain each number.

a. $\sqrt{9}$ b. $\sqrt{0}$ c. $-\sqrt{4}$

d. $\sqrt{\frac{4}{9}}$ e. $\sqrt{2}$ f. $\sqrt{-1}$

Exploration 1

Imaginary unit (i) defined as $i = \sqrt{-1}$

note $i^2 = -1$

The Imaginary unit i can be used to write the square root of any negative number

Work with a partner. Use the definition of the imaginary unit i to match each quadratic equation with its complex solution. Justify your answers.

a. $x^2 - 4 = 0$ b. $x^2 + 1 = 0$ c. $x^2 - 1 = 0$

d. $x^2 + 4 = 0$ e. $x^2 - 9 = 0$ f. $x^2 + 9 = 0$

A. i B. $3i$ C. 3

D. $2i$ E. 1 F. 2

$x^2 + 4 = 0$
$x^2 = -4$
$\sqrt{x^2} = \sqrt{-4}$
$x = 2i$

$x^2 + 9 = 0$
$x^2 = -9$
$\sqrt{y^2} = \sqrt{-9}$
$x = 3i$

Exploration 2

$x^2 - 4 = 0$
$x^2 = 4$
$\sqrt{x^2} = \sqrt{4}$
$x = 2$

$x^2 + 1 = 0$
$x^2 = -1$
$\sqrt{x^2} = \sqrt{-1}$
$x = i$

$x^2 - 1 = 0$
$x^2 = 1$
$\sqrt{x^2} = \sqrt{1}$
$x = 1$

$x^2 - 9 = 0$
$x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$
$x = 3$

Core Concept

The Square Root of a Negative Number

Property	**Example**
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

Core Concept

Standard form always puts the numerical value before the imaginary unit (i)

example: $\sqrt{-9} = 3i$
$\sqrt{-1} \cdot \sqrt{9}$
$i \cdot 3 = 3i$

Find the square root of each number.

a. $\sqrt{-25}$ b. $\sqrt{-72}$ c. $-5\sqrt{-9}$

$\sqrt{-1}\cdot\sqrt{25}$
$i\cdot 5$
$5i$

$\sqrt{-1}\cdot\sqrt{72}$
$2\ 36$
$\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{36}$
$i\cdot\sqrt{2}\cdot 6$
$6i\sqrt{2}$

$-5\sqrt{-1}\cdot\sqrt{9}$
$-5\ i\cdot 3$
$-15i$

Example 1

you can rewrite negatives under the radical by seperating things out. This will help you to keep your work organized.

Find the square root of the number.

1. $\sqrt{-4}$ 2. $\sqrt{-12}$ 3. $-\sqrt{-36}$ 4. $2\sqrt{-54}$

$\sqrt{-1}\cdot\sqrt{4}$
$i\cdot 2$
$2i$

$\sqrt{-1}\cdot\sqrt{12}$
$4\ 3$
$i\cdot 2\cdot\sqrt{3}$
$2i\sqrt{3}$

Monitoring Progress 1-4

Additional practice for students. Check your work and ask for help.

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

$\frac{2x}{2} = \frac{10}{2}$ $-7i = yi$

$x = 5$ $-7 = y$

Example 2

Complex number: Standard form:

$a + bi$

real part Imaginary part

* Set real parts equal
* Set Imaginary parts equal

Find the values of x and y that satisfy the equation.

5. $x + 3i = 9 - yi$

6. $9 + 4yi = -2x + 3i$

$x = 9$

$3i = -yi$

$-3 = y$

$9 = -2x$

$-\frac{9}{2} = x$

$4yi = 3i$

$y = \frac{3}{4}$

Monitoring Progress 5-6

* Set the real parts equal - solve for the variable

* Set imaginary parts equal, find the only value that will make the equation true.

Core Concept

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Core Concept

* Same idea as combining like terms

* Imaginary units can be combined just like variable terms - follow the same rules.

Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$

$(8 + 5) + (-i + 4i)$

$13 + 3i$

b. $(7 - 6i) - (3 - 6i)$

$(7 - 3) + (-6i + 6i)$

4

c. $13 - (2 + 7i) + 5i$

$13 - 2 - 7i + 5i$

$11 - 2i$

Example 3

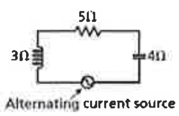
* always answer in standard form

$a + bi$

* watch signs - distribute negatives first to stay organized.

Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω, the uppercase Greek letter omega.

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	$-Ci$



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit.

Example 4

Impedance of Circuit

$5 + 3i + (-4i) = 5 - i$

$5 - i = 5 - i$

* Impedance of the circuit is $(5 - i)$ ohms

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

$4i(-6) + (4i \cdot i)$

$-24i + 4i^2$

$-24i + 4(-1)$

$-24i - 4$

$-4 - 24i$

b. $(9 - 2i)(-4 + 7i)$

	9	$-2i$
-4	-36	$8i$
$+7i$	$63i$	$-14i^2$ 14

$-36 + 14 + 8i + 63i$

$-32 + 71i$

Example 5

* to multiply two complex numbers, use the distributive property or the FOIL method.

* always state your final answer in standard form.

7. WHAT IF? In Example 4, what is the impedance of the circuit when the capacitor is replaced with one having a reactance of 7 ohms?

$(5 - 4i)$ ohms

Perform the operation. Write the answer in standard form.

8. $(9 - i) + (-6 + 7i)$ **9.** $(3 + 7i) - (8 - 2i)$ **10.** $-4 - (1 + i) - (5 + 9i)$

11. $(-3i)(10i)$ **12.** $i(8 - i)$ **13.** $(3 + i)(5 - i)$

Monitoring Progress 7-13

* Student practice - Check with me for questions and answers!

Solve (a) $x^2 + 4 = 0$ and (b) $2x^2 - 11 = -47$.

$x^2 + 4 = 0$

$x^2 = -4$

$\sqrt{x^2} = \pm\sqrt{-4}$

$x = \pm 2i$

$2x^2 - 11 = -47$

$+11 \quad +11$

$\frac{2x^2}{2} = \frac{-36}{2}$

$x^2 = -18$

$\sqrt{x^2} = \pm\sqrt{-18}$

$x = \pm i\sqrt{18}$

$x = \pm 3i\sqrt{2}$

Example 6

* Isolate the variable → use proper order of operations to undo the equation.

* even Index requires us to solve for the positive and negative solution

Find the zeros of $f(x) = 4x^2 + 20$.

$4x^2 + 20 = 0$

$\frac{4x^2}{4} = \frac{-20}{4}$

$x^2 = -5$

$\sqrt{x^2} = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

Example 7

* always Check your answer to make sure both solutions work for the given equation

Solve the equation.

14. $x^2 = -13$ **15.** $x^2 = -38$ **16.** $x^2 + 11 = 3$

17. $x^2 - 8 = -36$ **18.** $3x^2 - 7 = -31$ **19.** $5x^2 + 33 = 3$

Find the zeros of the function.

20. $f(x) = x^2 + 7$ **21.** $f(x) = -x^2 - 4$ **22.** $f(x) = 9x^2 + 1$

Monitoring Progress 14-22

* Student practice

I Used to Think ... But Now I Know: Take time for students to reflect on their current understanding of complex numbers.

Closure

* reflect on what you have learned. what questions do you still have?