

Algebra in Genetics: The Hardy-Weinberg Law

Some people have attached earlobes, the recessive trait. Some people have free earlobes, the dominant trait. What percent of people carry both traits?

Part 1: Background

Algebra is everywhere—even in your genes! A famous quadratic equation called the Hardy-Weinberg Law can predict the number of people in large populations who carry certain genetic combinations. These combinations are called genotypes.

What is a genotype? Human beings have thousands of genes that determine characteristics such as eye color, hair color, and height. Each gene has different versions called alleles. For example, an ear lobe gene would have an allele for free lobes, earlobes that hang below the point where they attach, and an allele for attached lobes. Humans carry two alleles, one from a mother and one from a father, for each gene. These two alleles together make up a genotype for a specific trait. The possible genotypes for free (F) and attached (a) earlobes are shown in the table.



Because one allele is physically dominant, it is difficult to observe how many people carry *both* alleles. For example, a person with free lobes could have the *FF* genotype, *or* he or she could have the *Fa* or *aF* genotypes. The only thing that is known for sure is that the person with attached lobes carries the *aa* genotype and does *not* have both alleles.

Part 2: Creating the Model

So how can you know how many people carry both alleles? That is where the algebra comes in!

Considering a single allele, let

- p = the probability that the single allele is F, dominant free lobes, and
- q = the probability that the single allele is *a*, recessive attached lobes.
- **a.** Because the allele is either one or the other, the chance of having one or the other is 100%. Write a sum that represents this.

____ + ____ = 1

b. However, you need to know something about two alleles. The probability of having both alleles is given by $p \bullet q$. What can you do to the binomial in part (a) to create a term with $p \bullet q$?

(_____+ ____)^- = 1^-__

c. Expand the quadratic equation. This equation is known as the Hardy-Weinberg Law.

_____ + ____ + ____ = 1

3 Performance Task (continued)

Part 3: Understanding the Model

Your model now has terms representing *genotypes*, two alleles. Use the fact that you multiply to find the probability of two independent events to answer the following.

a. What do the quadratic terms represent?

 $p^2 = p \bullet p$ represents

- $q^2 = q \bullet q$ represents
- **b.** What does the term 2pq represent?

Part 4: Using the Model

In a population of 10,000 people, suppose that 80% have free earlobes and 20% have attached earlobes. Remember, you can observe only their *physical* traits. How can you calculate the number of people who carry *both* alleles?

- **a.** Which genotypes have *both* alleles? Note that you are trying to find the percentage of the population that has these genotypes.
- **b.** Because free earlobes are dominant, which three genotypes comprise 80% of the population?
- **c.** Because attached earlobes are recessive, which genotype does 20% of the population have?
- **d.** Explain how you know that $q^2 = q \bullet q = 0.2$.
- **e.** Now that you know the numerical value of q^2 , what operation will you use to find the numerical value of q? Find q.
- **f.** Now that you know q, use the equation from part (2a) to find p.
- **g.** Find $2p \bullet q$, the probability of carrying both alleles. Genotypes with both alleles are *Fa* and *aF*.
- **h.** In a population of 10,000 people, how many carry *both* alleles?



Teacher Notes: