Give the coordinates of the image of point $P(-5, 3)$ after each reflection.

1. reflection in the y-axis
2. reflection in the x-axis
3. reflection in the line through $(5, -6)$ and $(8, -6)$
4. reflection in the line through $(-1, -1)$ and $(-1, -2)$

Warm Up

* Review of transformations
* Students should do for practice

Determine if the data show a linear relationship. If so, write an equation of a line of fit. Estimate y when $x = 20$ and explain its context in the situation.

1.

Minutes jogging, x	2	5	10	15
Calories burned, y	22	55	110	165

2.

Years, x	10	12	17	21
Height (feet), y	4.2	5.0	6.0	6.1

Cumulative Warm Up

* discuss how we determine if we have a linear relationship or not
* Review concept

Essential Question

What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

- find vertex (h, k)
- and as points move away from the vertex they are equal distance. (y-values)

Essential Question

What you will learn:

- Explore properties of parabolas
- Find maximum or minimum values of quadratic functions
- Graph Quadratics using X-Intercepts
- Solve real world problems.

Work with a partner.

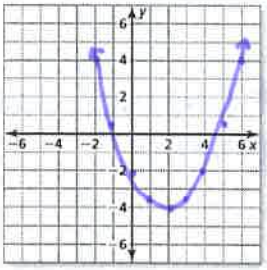
a. Complete the table. Then use the values in the table to sketch the graph of the function

$f(x) = \frac{1}{2}x^2 - 2x - 2$

on graph paper.

x	−2	−1	0	1	2
$f(x)$	4	.5	-2	-3.5	-4

x	3	4	5	6
$f(x)$	-3.5	-2	.5	4



Exploration 1a

- Use table to help calculate points to graph the parabola.
- Notice the y-values as they move away from the vertex

b. Use the results in part (a) to identify the vertex of the parabola.

c. Find a vertical line on your graph paper so that when you fold the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?

d. Show that the vertex form

$f(x) = \frac{1}{2}(x-2)^2 - 4$

is equivalent to the function given in part (a).

$\frac{1}{2}(x-2)(x-2) - 4$

$\frac{1}{2}(x^2 - 4x + 4) - 4$

$\frac{1}{2}x^2 - 2x + 2 - 4 \rightarrow \frac{1}{2}x^2 - 2x - 2$

Exploration 1b-d

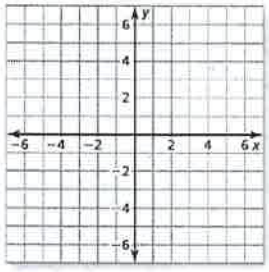
b.) (2, -4)

c.) $x = 2$: It is the x coordinate of the vertex.

d.) $f(x) = a(x-h)^2 + k$
We can calculate to write in standard form
$y = ax^2 + bx + c$

Work with a partner. Repeat Exploration 1 for the function given by

$f(x) = -\frac{1}{3}x^2 + 2x + 3 = -\frac{1}{3}(x-3)^2 + 6$



Exploration 2

- leave as practice for Students
- follow same steps as Exploration 1.

$a=-2$ $h=-3$ $k=4$

Graph $f(x) = -2(x + 3)^2 + 4$. Label the vertex and axis of symmetry.

$(-3, 4)$ ← vertex

axis of symmetry is always the x-coordinate of the vertex

axis of symmetry = -3

Example 1

- begin with vertex form

$f(x) = a(x-h)^2+k$

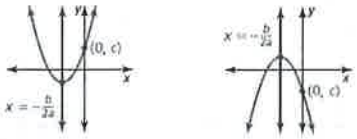
- have students find the vertex

- discuss how we would graph by hand once we have the vertex.

Core Concept

Properties of the Graph of $f(x) = ax^2 + bx + c$

$y = ax^2 + bx + c, a > 0$ $y = ax^2 + bx + c, a < 0$



- The parabola opens up when $a > 0$ and opens down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

Core Concept

Standard form

$y = ax^2 + bx + c$

- have students label points of emphasis on their own graphs.

Graph $f(x) = 3x^2 - 6x + 1$. Label the vertex and axis of symmetry.

$a=3$ $b=-6$ $c=1$

$x = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$ ← axis of symmetry

$y = 3(1)^2 - 6(1) + 1$

$y = 3 - 6 + 1$

$y = -3 + 1$

$y = -2$

vertex $(1, -2)$

Example 2

① axis of symmetry

$x = \frac{-b}{2a}$

② Substitute axis of symmetry into the original equation to find the vertex.

Graph the function. Label the vertex and axis of symmetry.

1. $f(x) = -3(x + 1)^2$

(-1, 0) (V)

x = -1 (Axis)

2. $g(x) = 2(x - 2)^2 + 5$

(V) = (2, 5)

(Axis) = x = 2

3. $h(x) = x^2 + 2x - 1$

$\frac{-2}{2(1)} = \frac{-2}{2} = -1$

$(-1)^2 + 2(-1) - 1$

$1 - 2 - 1$

-2

(-1, -2)

4. $p(x) = -2x^2 - 8x + 1$

$\frac{8}{2(-2)} = \frac{8}{-4} = -2$

$-2(-2)^2 - 8(-2) + 1$

$-8 + 16 + 1$

9

(-2, 9)

Monitoring Progress 1-4

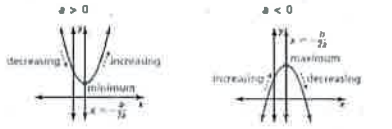
Student practice

* need to know both methods.

Core Concept

Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.



- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

Core Concept

- using a term to see minimum and maximum values.
- Domain and Range

Find the minimum value or maximum value of $f(x) = \frac{1}{2}x^2 - 2x - 1$.

Describe the domain and range of the function, and where the function is increasing and decreasing.

$x = \frac{-(-2)}{2(\frac{1}{2})} = 2$

$\frac{1}{2}(2)^2 - 2(2) - 1$

$2 - 4 - 1$

$-2 - 1 = -3$

(2, -3)

Example 3

a > 0 min. value

domain: all real #'s

Range $y \geq -3$

Use graphing Calculator to describe increase/decrease

decrease left of x = 2

increase right of x = 2

5. Find the minimum value or maximum value of (a) $f(x) = 4x^2 + 16x - 3$ and (b) $h(x) = -x^2 + 5x + 9$. Describe the domain and range of each function, and where each function is increasing and decreasing.

Monitoring Progress 5

* Student practice

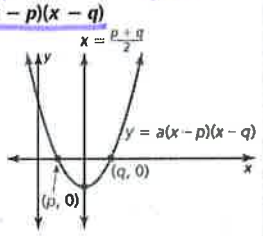
- Make sure students are doing each part!
- Can use calculator!

Core Concept

Intercept form

Properties of the Graph of $f(x) = a(x - p)(x - q)$

- Because $f(p) = 0$ and $f(q) = 0$, p and q are the x-intercepts of the graph of the function.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The parabola opens up when $a > 0$ and opens down when $a < 0$.



Core Concept

- Remember x-intercept means the graph is crossing the x-axis at that point so y must equal 0.

Graph $f(x) = -2(x + 3)(x - 1)$. Label the x-intercepts, vertex, and axis of symmetry.

Axis of symmetry

$x = \frac{p+q}{2} = \frac{-3+1}{2} = \frac{-2}{2} = -1$

Vertex

$y = -2(-1+3)(-1-1)$
$= -2(2)(-2)$
$= 8$
$(-1, 8)$

Example 4

$p = -3 \quad q = 1$

x-Intercepts $(-3, 0)$ and $(1, 0)$

- Use information to graph
  - plot points
  - sketch parabola

Graph the function. Label the *x*-intercepts, vertex, and axis of symmetry.

6. $f(x) = -(x + 1)(x + 5)$

7. $g(x) = \frac{1}{4}(x-6)(x-2)$

6)

$a = 1$

$p = -1$

$q = -5$

$(-1, 0)$

$(-5, 0)$

$x = \frac{-1 + -5}{2} = \frac{-6}{2} = -3$

$-(-3+1)(-3+5)$

$-(-2)(2)$

4

$(-3, 4)$

Monitoring Progress 6-7

7.) $a = 1/4$

$p = 6$ $(6, 0)$

$q = 2$ $(2, 0)$

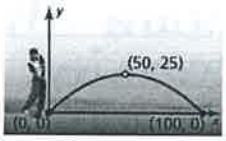
$x = \frac{6+2}{2} = \frac{8}{2} = 4$

$\frac{1}{4}(4-6)(4-2)$

$\frac{1}{4}(-2)(2)$ $(4, -1)$

$\frac{1}{4}(-4) = -1$

The parabola shows the path of your first golf shot, where x is the horizontal distance (in yards) and y is the corresponding height (in yards). The path of your second shot can be modeled by the function $f(x) = -0.02x(x - 80)$. Which shot travels farther before hitting the ground? Which travels higher?



$f(x) = -.02(x-0)(x-80)$

$x = \frac{p+q}{2} = \frac{0+80}{2} = 40$

$f(40) = -0.02(40)(40-80) = 32$

max height second shot = 32

$100 > 80$ 1st shot travels further

$32 > 25$ and shot higher.

Example 5

X-Intercepts = 0 and 100

- ball travels 100 yards before hitting the ground
- Axis of symmetry 1/2 way between (0, 0) and (100, 0)

$\frac{0+100}{2} = 50$

$x = 50$ (50, 25) vertex

8. WHAT IF? The graph of your third shot is a parabola through the origin that reaches a maximum height of 28 yards when $x = 45$. Compare the distance it travels before it hits the ground with the distances of the first two shots.

Monitoring Progress 8

* Student practice

Writing Prompt: To graph $f(x) = ax^2 + bx + c$, you ...

Closure