

Perform the indicated operation. Simplify your answer and indicate any excluded values of x .

$$1. \frac{2}{7} \cdot \frac{5}{3} = \frac{10}{21}$$

$$2. \frac{9}{10} \cdot \frac{8}{5} = \frac{72}{50} = \frac{36}{25}$$

$$3. \frac{4}{11} \div \frac{2}{3} = \frac{4}{11} \cdot \frac{3}{2} = \frac{12}{22} = \frac{6}{11}$$

$$4. \frac{x-1}{5} \cdot \frac{2}{5} = \frac{2(x-1)}{25}$$

$$5. \frac{x}{x-1} \cdot \frac{x}{x+1} = \frac{x^2}{(x-1)(x+1)}$$

$$6. \frac{-3}{2x+1} + \frac{x}{x+1} = \frac{-3(x+1)}{x(2x+1)} + \frac{x(x+1)}{x(2x+1)} = \frac{-3(x+1) + x(x+1)}{x(2x+1)}$$

Warm Up

You deposit \$7000 in an account that pays annual interest. Find the balance in the account after 3 years if the interest is compounded as described below at the given rate.

- 1. quarterly; 1.26%
- 2. monthly; 3.25%
- 3. daily; 2.8%
- 4. continuously; 2.3%
- 5. annually; 4.125%
- 6. continuously; 1.86%

Cumulative Warm Up

Essential Question

How can you determine the excluded values in a product or quotient of two rational expressions?

Essential Question

When multiplying - do you need a common denominator?

How does multiplication change if there are variables? Does it change?

What you will learn:

Work with a partner. Find the product or quotient of the two rational expressions. Then match the product or quotient with its excluded values. Explain your reasoning.

Product or Quotient Excluded Values

a. $\frac{1}{x-1} \cdot \frac{x-2}{x+1} =$

A. -1, 0, and 2

b. $\frac{1}{x-1} \cdot \frac{-1}{x-1} =$

B. -2 and 1

c. $\frac{1}{x-2} \cdot \frac{x-2}{x+1} =$

C. -2, 0, and 1

Exploration 1a-c

d. $\frac{x+2}{x-1} \cdot \frac{-x}{x+2} =$

D. -1, and 2

e. $\frac{x}{x+2} + \frac{x+1}{x+2} =$

E. -1, 0, and 1

f. $\frac{x}{x-2} + \frac{x+1}{x} =$

F. -1 and 1

g. $\frac{x}{x+2} + \frac{x}{x-1} =$

G. -2 and -1

Exploration 1 d-g

h. $\frac{x+2}{x} + \frac{x+1}{x-1} =$

H. 1

Exploration 1 h

always keep in mind
the denominator can not
be equal to zero.

Work with a partner. Write a product or quotient of rational expressions that has the given excluded values. Justify your answer.

- a. -1 b. -1 and 3 c. -1, 0, and 3

Exploration 2

Core Concept

Simplifying Rational Expressions

Let a , b , and c be expressions with $b \neq 0$ and $c \neq 0$.

Property $\frac{ac}{bc} = \frac{a}{b}$ Divide out common factor c .

Examples $\frac{15}{65} = \frac{3 \cdot 5}{13 \cdot 5} = \frac{3}{13}$ Divide out common factor 5.

$\frac{4(x+3)}{(x+3)(x+3)} = \frac{4}{x+3}$ Divide out common factor $x+3$.

Core Concept

Simplify $\frac{x^2 - 4x - 12}{x^2 - 4}$

$$\frac{(x+2)(x-6)}{(x+2)(x-2)}$$

$$\frac{\cancel{(x+2)}(x-6)}{\cancel{(x+2)}(x-2)} = \frac{x-6}{x-2}$$

Example 1

• begin by factoring the numerator and the denominator

• any number divided by itself equals 1.

• Always state what x can not equal

$$x \neq 2, -2$$

Simplify the rational expression, if possible.

1. $\frac{2(x+1)}{(x+1)(x+3)}$ 2. $\frac{x+4}{x^2-16}$ $\frac{(x+4)}{(x-4)(x+4)}$

$\frac{2}{x+3}$ $x \neq -1, -3$ $\frac{1}{x-4}$ $x \neq -4, 4$

3. $\frac{4}{x(x+2)}$ 4. $\frac{x^2-2x-3}{x^2-x-6}$ $\frac{(x+1)(x-3)}{(x+2)(x-3)} = \frac{x+1}{x+2}$

Simplified $x \neq 0, x \neq -2$ $x \neq -2, 3$

Monitoring Progress 1-4

Core Concept

Multiplying Rational Expressions
 Let $a, b, c,$ and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

Example $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{10 \cdot 3 \cdot x^2 \cdot x^2 \cdot y^3}{10 \cdot 2 \cdot x \cdot y^3} = \frac{3x^2}{2}, x \neq 0, y \neq 0$

Core Concept

Find the product $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$

$\frac{8 \cdot 7 \cdot \overset{x^2}{\cancel{x^3}} \cdot x^4 \cdot y \cdot \overset{y^2}{\cancel{y^3}}}{8 \cdot \cancel{x} \cdot \cancel{y^2} \cdot \cancel{y^3} \cdot y}$

$7x^6y$

Example 2

• Cross simplifying : or remembering you can simplify a top and a bottom.

• you can always multiply straight across the top and straight across the bottom and simplify at the end.

Find the product $\frac{3x-3x^2}{x^2+4x-5} \cdot \frac{x^2+x-20}{3x}$

$$\frac{-3x(x-1)}{(x+5)(x-1)} \cdot \frac{(x+5)(x-4)}{3x}$$

$$\frac{-\cancel{3x}(x-1)}{(x+5)\cancel{(x-1)}} \cdot \frac{(x+5)\cancel{(x-4)}}{3x}$$

$-1(x-4)$ or $-x+4$

$x \neq -5, 1, 0$

Example 3

• look for GCF's in numerators and denominators.

Find the product $\frac{x+2}{x^2-27} \cdot (x^2+3x+9)$

$$\frac{x+2}{(x-3)\cancel{(x^2+3x+9)}} \cdot \cancel{x^2+3x+9}$$

$$\frac{x+2}{x-3}$$

Example 4

check for special case scenarios when factoring

Find the product.

5. $\frac{3x^2y^2}{8xy} \cdot \frac{6xy^2}{9x^2y}$ 6. $\frac{2x^2-10x}{x^2-25} \cdot \frac{x+3}{2x^2}$ 7. $\frac{x+5}{x^3-1} \cdot (x^2+x+1)$

Monitoring Progress 5-7

* Student practice

Core Concept

Dividing Rational Expressions

Let $a, b, c,$ and d be expressions with $b \neq 0, c \neq 0,$ and $d \neq 0.$

Property $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ Simplify $\frac{ad}{bc}$ if possible.

Example $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)} \cdot x \neq \frac{3}{2}$

Core Concept

how is division different from multiplying?

Find the quotient $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$

$$\frac{7x}{2(x-5)} \cdot \frac{(x^2-11x+30)}{x(x-6)}$$

$$\frac{7x}{2(x-5)} \cdot \frac{\cancel{(x-6)}\cancel{(x-5)}}{x\cancel{(x-6)}}$$

$x \neq 5, 0, 6$

Example 5

- remember: keep it, change it, flip it!
- follow the same steps factor and then simplify.

Find the quotient $\frac{6x^2+x-15}{4x^2} \div (3x^2+5x)$

$6 \cdot 15 = 90$
 $-9 + 10$

$$\frac{6x^2+x-15}{4x^2} \cdot \frac{1}{3x^2+5x}$$

$$\frac{\cancel{(3x+5)}\cancel{(2x-3)}}{4x^2} \cdot \frac{1}{x\cancel{(3x+5)}}$$

$x \neq 0, -5/3$

$$\frac{2x-3}{4x^2}$$

Example 6

$$\frac{(6x^2-9x)(10x-15)}{3x(2x-3)+5(2x-3)}$$

$$(3x+5)(2x-3)$$

- remember the ac method
- find products
- find sums
- write polynomial in 4 terms and factor by grouping.

The total annual amount I (in millions of dollars) of personal income earned in Alabama and its annual population P (in millions) can be modeled by

$$I = \frac{6922t + 106,947}{0.0063t + 1}$$

and $P = 0.0343t + 4.432$

where t represents the year, with $t = 1$ corresponding to 2001. Find a model M for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume $t > 0$.)

Example 7

Find the quotient.

8. $\frac{4x}{5x-20} \div \frac{x^2-2x}{x^2-6x+8}$

9. $\frac{2x^2+3x-5}{6x} \div (2x^2+5x)$

* Student practice

Monitoring Progress 8-9

Exit Ticket: Multiply the expressions and simplify the result:

$$\frac{4x+20}{x^2} \cdot \frac{x^2+x}{2x+10}$$

Closure

