Determine the value of n in the expression.

1. $\frac{3^n}{3^2} = 3^5$
2. $\frac{x^4}{x^n} = x^{-2}$
3. $x^2 x^n = x^{5/2}$
4. $(2^n)^n = 4$
5. $\frac{(y^n)^2}{y^3} = y$
6. $y^{2n} y = y^{-5}$

$n - 2 = 5$
$+2 \quad +2$
$n = 7$

$3^{n-2} = 3^5$

$x^{4-n} = x^{-2}$
$4 - n = -2$
$-4 \quad\quad -4$
$-n = -6$
$n = 6$

$x^{2+n} = x^{5/2}$

$2+n = \frac{5}{2} - \frac{2}{1}$
-2
$n = \frac{5}{2} - \frac{4}{2}$
$n = \frac{1}{2}$

2

Warm Up

• Remember when your base value is the same your exponent values are equal to each other.

• If bases are not the same ask yourself, "Is there a way I can rewrite this so the bases are the same?"

• Remember exponent rules!

Solve the equation. Write your answer in simplest form.

1. $2(x-3)^2 = 8$
2. $6x^2 - 7x - 20 = 0$
3. $x^2 - 4x - 8 = 0$
4. $6x - 3x^2 = 15$
5. $x^2 = 6x - 9$
6. $-\frac{1}{2}(3x+1)^2 = -5$

Cumulative Warm Up

* Spiral review *

Student practice solving

Essential Question

How can you use properties of exponents to derive properties of logarithms?

* how do we write exponents from logs?

$\log_b m = x \rightarrow b^x = m$

$\log_b n = y \rightarrow b^y = n$

Essential Question

What you will learn:

• Use properties of logarithms to evaluate logarithms

• Use properties of logarithms to expand or condense logarithms expressions

• Use the change-of-base formula to evaluate logarithms.

Work with a partner. To derive the Product Property, multiply m and n to obtain $mn = b^x b^y = b^{x+y}$.

The corresponding logarithmic form of $mn = b^{x+y}$ is $\log_b mn = x + y$. So,

$\log_b mn =$. Product Property of Logarithms

$\log_b m + \log_b n$

Exploration 1

Use Product of Powers property

$a^m \cdot a^n = a^{m+n}$

*remember b^{x+y} is the same as $b^x \cdot b^y$

$\log_b x + \log_b y$

Work with a partner. To derive the Quotient Property, divide m by n to obtain

$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$.

The corresponding logarithmic form of $\frac{m}{n} = b^{x-y}$ is $\log_b \frac{m}{n} = x - y$. So,

$\log_b \frac{m}{n} =$. Quotient Property of Logarithms

Exploration 2

Work with a partner. To derive the Power Property, substitute bx for m in the expression $\log_b m^n$, as follows.

$\log_b m^n = \log_b (b^x)^n$	Substitute b^x for m.
$= \log_b b^{nx}$	Power of a Power Property of Exponents
$= nx$	Inverse Property of Logarithms

So, substituting $\log_b m$ for x, you have

$\log_b m^n =$. Power Property of Logarithms

Exploration 3

Core Concept

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Make sure to discuss exponent moving

Core Concept

• Create a table showing log rule and ~~its~~ its corresponding

$a^m a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{m \cdot n}$

Use $\log_2 3 \approx 1.585$ and $\log_2 7 \approx 2.807$ to evaluate each logarithm.

a. $\log_2 \frac{3}{7}$ **b.** $\log_2 21$ **c.** $\log_2 49$

a. $\log_2 3 - \log_2 7$

$\approx 1.585 - 2.807$

≈ -1.222

c. $\log_2 7^2$

$2(\log_2 7)$

$\approx 2(2.807)$

$= 5.614$

Example 1

b) $\log_2 21 = \log_2 (3 \cdot 7)$

$\log_2 3 + \log_2 7$

$\approx 1.585 + 2.807$

≈ 4.392

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

1. $\log_6 \frac{5}{8}$ **2.** $\log_6 40$

3. $\log_6 64$ **4.** $\log_6 125$

3. $\log_6 8^2$

$2 \log_6 8$

$2(1.161)$

2.694

4. $\log_6 5^3$

$3 \log_6 5$

$3(0.898)$

2.694

Monitoring Progress 1-4

1.) $\log_6 5/8 = \log_6 5 - \log_6 8$

$\approx 0.898 - 1.161$

≈ -0.263

2.) $\log_6 40 = \log_6 (5 \cdot 8)$

$= \cancel{(0.898) +}$

$\log_6 5 + \log_6 8$

$= (0.898) + 1.161$

$= 2.056$

* additional practice * from textbook

Expand $\ln \frac{5x^7}{y}$. $= \ln 5x^7 - \ln y$

$= \ln 5 + \ln x^7 - \ln y$

$= \ln 5 + 7\ln x - \ln y$

Example 2

* when expanding OR condensing an expression involving logarithms, you can assume that any variables are positive

ln = natural log.

Condense log 9 + 3 log 2 − log 3.

$\log 9 + \log 2^3 - \log 3$

$\log(9 \cdot 2^3) - \log 3$

$\log \frac{9 \cdot 2^3}{3}$

$\log 3 \cdot 2^3$

$\log 3 \cdot 8$

$\log 24$

Example 3

- Use exponent power rules
- Simplify as much as possible.

Expand the logarithmic expression.

5. $\log_6 3x^4$ **6.** $\ln \frac{5}{12x}$

$\log_6 3 + 4\log_6 x$ $\ln 5 - \ln 12 - \ln x$

Condense the logarithmic expression.

7. $\log x - \log 9$ **8.** $\ln 4 + 3\ln 3 - \ln 12$

$\log \frac{x}{9}$ $\ln 9$

Monitoring Progress 5-8

Student practice

Core Concept

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

- Common log → base 10
- natural log → base e
- basically have different bases and matter in higher math | programming | Science classes.

Core Concept

- the change of base formula enables you to rewrite logarithms with other bases as natural or common logarithms. Then a calculator can be used to evaluate or graph the expression.

Evaluate $\log_3 8$ using common logarithms.

$$\log_3 8 = \frac{\log 8}{\log 3}$$

$$= \frac{.09031}{.4771} \approx 1.893$$

$$\log_c a = \frac{\log a}{\log c}$$

Example 4

- Once in change of base formula use Calculator.
- Natural logs and common logs are interchangable once in change of base formula.

Evaluate $\log_6 24$ using natural logarithms.

$$\log_6 24 = \frac{\ln 24}{\ln 6}$$

$$= \frac{3.1781}{1.7918}$$

$$= 1.774$$

$$\log_c a = \frac{\ln a}{\ln c}$$

Example 5

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the intensity of the sound doubles. By how many decibels does the loudness increase?

Example 6

Use the change-of-base formula to evaluate the logarithm.

9. $\log_5 8$ **10.** $\log_8 14$ **11.** $\log_{26} 9$ **12.** $\log_{12} 30$

13. WHAT IF? In Example 6, the artist turns up the volume so that the intensity of the sound triples. By how many decibels does the loudness increase?

Monitoring Progress 9-13

• **Response Logs:** "I am feeling good about …" or "A sticky part for me is …" or "Right now I am thinking about …."

Closure