Determine if the model represents *exponential growth* or *exponential decay*. Then identify the initial amount and the percent increase or decrease.

1. $y = 1.2^x$
2. $y = 0.78^x$
3. $y = \left(\frac{5}{8}\right)^x$
4. $y = 28 + 1.03^x$
5. $y = 25{,}000 + 0.95^x$
6. $y = 2^x$

Warm Up

- Review
- Exponential growth
  $x > 1$
- Exponential decay
  $0 < x < 1$
- How can we do this if we don't remember

Find the inverse of the function. Then graph the function and its inverse in the same coordinate plane.

1. $y = \frac{1}{2}x - 5$
2. $y = -3x + 7$
3. $y = \frac{x-9}{5}$
4. $y = (x + 2)^3$
5. $y = \frac{x^3 - 6}{2}$
6. $y = -2x^2,\ x \geq 0$

Cumulative Warm Up

- parent function
- review how these transform.

Essential Question

What is the natural base *e*?

Natural base e → denoted by *e*. The natural base *e* is irrational → so we can not find it's ~~actual~~ exact amount

Essential Question

what you will learn:

- define and use natural base e
- Graph natural base e functions
- solve real-life problems.

Work with a partner. One way to approximate the natural base e is to approximate the sum

$$1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{1\cdot 2\cdot 3\cdot 4}+\cdots$$

Use a spreadsheet or a graphing calculator to approximate this sum. Explain the steps you used. How many decimal places did you use in your approximation?

Exploration 1

Work with a partner. Another way to approximate the natural base e is to consider the expression

$$\left(1+\frac{1}{x}\right)^x.$$

As x increases, the value of this expression approaches the value of e. Copy and complete the table. Then use the results in the table to approximate e. Compare this approximation to the one you obtained in Exploration 1.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{x}\right)^x$						

Exploration 2

Work with a partner. Use your approximate value of e in Exploration 1 or 2 to complete the table. Then sketch the graph of the *natural base exponential function* $y = e^x$. You can use a graphing calculator and the e^x key to check your graph. What are the domain and range of $y = e^x$? Justify your answers.

x	-2	-1	0	1	2
$y = e^x$					

Exploration 3

Core Concept

The Natural Base e

The natural base e is irrational. It is defined as follows:

As x approaches $+\infty$, $\left(1+\frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{x}\right)^x$	2.59374	2.70481	2.71692	2.71692	2.71827	2.71828

Core Concept

Simplify each expression.

a. $e^3 \cdot e^6$ b. $\frac{16e^5}{4e^4}$ c. $\left(3e^{-4x}\right)^2$

e^{3+6} $4e^{5-4}$ $3^2(e^{-4x})^2$

e^9 $4e^1$ $9e^{-8x}$

$4e$ $\frac{9}{e^{8x}}$

Example 1

- Use graphing calculator to complete when solving.
- review exponent rules so students can simplify natural base e.

Student practice

Simplify the expression.

1. $e^7 \cdot e^4$ 2. $\frac{24e^8}{8e^5}$ 3. $\left(10e^{-3x}\right)^3$

e^{7+4} $4e^{8-5}$ $(10)^3(e^{-3x})^3$

e^{11} $4e^3$ $\frac{1000}{e^{9x}}$

Monitoring Progress 1-3

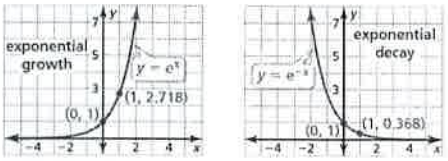
Core Concept

Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.



Core Concept

Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $y = 3e^x$ **b.** $f(x) = e^{-0.5x}$

growth

x	y
-2	.41
-1	1.10
0	3
1	8.15

decay

x	y
-4	7.39
-2	2.72
0	1
2	.37

Example 2

- Use graphing calculator to enter these and graph
- Use table function to see table values.

Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

4. $y = \frac{1}{2}e^x$ **5.** $y = 4e^{-x}$ **6.** $f(x) = 2e^{2x}$

growth decay growth

Monitoring Progress 4-6

- Student practice - guided → use graphing calculator

Core Concept

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

Core Concept

Compound Interest formula

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

· as the frequency of n compounding approaches positive infinity, the compound interest formula approximates

$A = Pe^{rt}$

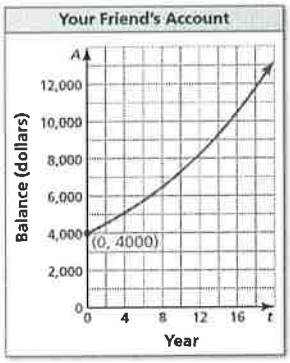
$P = 4500 \qquad t = 10$

$A = 4500e^{.04t}$

$A = 4500e^{.04(10)}$

$A = \$6713.21$

You and your friend each have accounts that earn annual interest compounded continuously. The balance A (in dollars) of your account after t years can be modeled by $A = 4500e^{0.04t}$. The graph shows the balance of your friend's account over time. Which account has a greater principal? Which has a greater balance after 10 years?



Example 3

$A = Pe^{rt}$

$A = 4250e^{.05(10)}$

$A = 7,007.07$

Show on graphing calculator

7. You deposit \$4250 in an account that earns 5% annual interest compounded continuously. Compare the balance after 10 years with the accounts in Example 3.

Monitoring Progress 7

- **Response Logs:** "Right now I am thinking about ..." or "Tomorrow I need to find out ..."

Closure