Notes Section 5.5

Simplify.

1. $3x(x^3+2x)$ 2. $\frac{14x}{-2x^7}$ 3. $\frac{x^3}{x^2-x}$

$3x^4+6x^2$ $-\frac{7}{x^6}$ $\frac{x^2}{x-1}$

4. $(ab)^4$ 5. $(a+b)(a-6b)$ 6. $(x-7x+6)(x)$

a^4b^4 $a^2-5ab-6b^2$ $-6x^2+6x$

Warm Up

use - distributive property
- exponent property rules
- combine like terms.

* show each step

Essential Question

How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

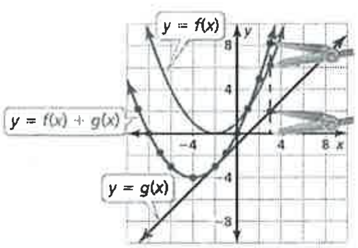
Essential Question

What you will learn:

* add, subtract, multiply and divide functions.

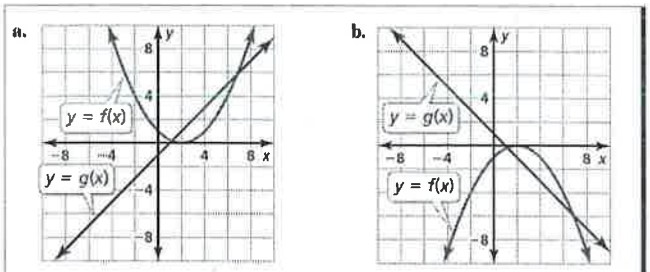
Work with a partner. Use the graphs of f and g to sketch the graph of $f+g$. Explain your steps.

Sample Use a compass or a ruler to measure the distance from a point on the graph of g to the x-axis. Then add this distance to the point with the same x-coordinate on the graph of f. Plot the new point. Repeat this process for several points. Finally, draw a smooth curve through the new points to obtain the graph of $f+g$.



Exploration 1

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication and division to form other real numbers, two functions can be combined to form other functions.



Exploration 1a-b

additional Graphs

* write equations for both graphs and then Combine those graph (add)

Core Concept

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f+g)(x) = f(x) + g(x)$	$(f+g)(x) = 5x + (x+2) = 6x + 2$
Subtraction	$(f-g)(x) = f(x) - g(x)$	$(f-g)(x) = 5x - (x+2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x+2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x+2}$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

Core Concept

functions are stated as
$f(x)$ = f of x
$g(x)$ = g of x

Operations on functions has us adding, subtracting multiplying or dividing functions together.

Let $f(x) = 3\sqrt{x}$ and $g(x) = -10\sqrt{x}$. Find $(f+g)(x)$ and state the domain. Then evaluate the sum when $x = 4$.

$f(x) = 3\sqrt{x}$ $\quad (f+g)(x)$

$g(x) = -10\sqrt{x}$ $\quad 3\sqrt{x} + (-10\sqrt{x})$

$-7\sqrt{x}$

evaluate the sum when $x = 4$

$(f+g)(4) = -7\sqrt{4} = -7 \cdot 2 = -14$

Example 1

* begin by combining the two functions
 - combine like terms as needed

* when given a sum for the variable - finish by substituting in the combined function and solving.

Notes Section 5.5

Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate the difference when $x = -2$.

$$(f-g)(x) = (3x^3-2x^2+5) - (x^3-3x^2+4x-2)$$

$$(3x^3-2x^2+5-x^3+3x^2-4x+2)$$

$$2x^3+x^2-4x+7$$

$$(f-g)(-2) = 2(-2)^3+(-2)^2-4(-2)+7$$

$$=2(-8)+4+8+7$$

$$=-16+12+7$$

$$=3$$

Example 2

* Combine the two functions
- follow rules for combining like terms

* Substitute the value for x into the combined function and solve.

** Watch signs **

Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate the product when $x = 9$.

$$(fg)(x) = x^2 \cdot \sqrt{x}$$

$$= x^2 \cdot x^{\frac{1}{2}}$$

$$= x^{(2+\frac{1}{2})} = x^{(\frac{4}{2}+\frac{1}{2})}$$

$$= x^{5/2}$$

$$(fg)(9) = 9^{5/2} = (\sqrt{9})^5 = 3^5$$

$$= 243$$

Example 3

Multiplication

Combine both functions

* all rules apply - especially exponent rules

Substitute given value for the variable.

Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate the quotient when $x = 16$.

$$\left(\frac{f}{g}\right)(x) = \frac{6x}{x^{3/4}} = \frac{6x\,(\sqrt[4]{x})}{\sqrt[4]{x^3}\,(\sqrt[4]{x})}$$

$$= \frac{6x\sqrt[4]{x}}{x} = 6\sqrt[4]{x}$$

$$\left(\frac{f}{g}\right)(16) = 6\sqrt[4]{16} = 6 \cdot 2 = 12$$

Example 4

Division

* you can either convert rational exponents to radical expressions or you can leave and work with rational exponents

* Same rules are followed

Notes Section 5.5

1. Let $f(x) = -2x^{2/3}$ and $g(x) = 7x^{2/3}$. Find $(f+g)(x)$ and $(f-g)(x)$ and state the domain of each. Then evaluate $(f+g)(8)$ and $(f-g)(8)$.

$(f+g)(x) = -2x^{2/3} + 7x^{2/3} = 5x^{2/3}$

$(f+g)(8) = 5(8)^{2/3} = 5(2^3)^{2/3}$

$= 5(2)^2$

$= 5(4) = 20$

$(f-g)(x) = -2x^{2/3} - 7x^{2/3} = -9x^{2/3}$

$(f-g)(8) = -9(8)^{2/3}$

$= -9(2^3)^{2/3}$

$= -9(2^2)$

$= -9(4) = -36$

2. Let $f(x) = 3x$ and $g(x) = x^{1/5}$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate $(fg)(32)$ and $\left(\frac{f}{g}\right)(32)$.

$(fg)(x) = 3x \cdot x^{1/5}$

$3x^{5/5 + 1/5}$

$3x^{6/5}$

$\left(\frac{f}{g}\right)(x) = \frac{3x}{x^{1/5}} = 3x^{5/5 - 1/5}$

$= 3x^{4/5}$

$(fg)(32) = 3(32)^{6/5}$

$= 3(2^5)^{6/5}$

$= 3(2)^6$

$= 3(64)$

$= 192$

$\left(\frac{f}{g}\right)(32) = 3(32)^{4/5}$

$= 3(2^5)^{4/5}$

$= 3(2)^4$

$= 3(16)$

$= 48$

Monitoring Progress 1-2

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{9 - x^2}$. Use a graphing calculator to evaluate $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ when $x = 2$. Round your answers to two decimal places.

$(f+g)(x) = \sqrt{x} + \sqrt{9-x^2}$

To enter on a calculator:

- y=
- y_1 = enter function
- y_2 = enter function
- go back to main screen
- vars
- right arrow to y-vars
- Choose function (enter)
- Choose function Y_1 (enter)
- parenthesis (2)
- operation
- repeat for next value
- enter for answer

Example 5

For a white rhino, heart rate r (in beats per minute) and life span s (in minutes) are related to body mass m (in kilograms) by the functions

$r(m) = 241m^{-0.25}$ and $s(m) = (6 \times 10^6)m^{0.2}$.

a. Find $(rs)(m)$.

b. Explain what $(rs)(m)$ represents.

Example 6

Notes Section 5.5

3. Let $f(x) = 8x$ and $g(x) = 2x^{5/6}$. Use a graphing calculator to evaluate $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

4. In Example 5, explain why you can evaluate $(f+g)(3)$, $(f-g)(3)$, and $(fg)(3)$ but not $\left(\frac{f}{g}\right)(3)$.

5. Use the answer in Example 6(a) to find the total number of heartbeats over the lifetime of a white rhino when its body mass is 5.7×10^5 kilograms.

Monitoring Progress 3-5

• Exit Ticket: Let *f*(*x*) = 4*x* and *g*(*x*) = −2*x*$^{1/2}$. Find (*f* + *g*)(*x*), (*f* − *g*)(*x*), and (*fg*)(*x*). Evaluate each new function at *x* = 4.

Closure