Simplify.

1. $x^{-1} = \frac{1}{x}$
2. $3x^{-6} = \frac{3}{x^6}$
3. $\frac{4}{b^{-7}} = 4b^7$
4. $(ab^4)^1 = ab^4$
5. $\frac{y^6}{y^4} = y^2$
6. $\frac{p^4}{p} = p^3$

Warm Up

x^{-1} → can't have a negative exponent → switch location to make positive
$3x^{-6}$ → remember the exponent only affects the base - the value right in front
$\frac{y^6}{y^4}$ → when the bases are the same, you keep the base and subtract the exponent.

Essential Question

How can you use properties of exponents to simplify products and quotients of radicals?

Essential Question

We are going to find that our property rules for exponents are used in exactly the same way for rational exponents and radicals.

Work with a partner. Let *a* and *b* be real numbers. Use the properties of exponents to complete each statement. Then match each completed statement with the property it illustrates.

Statement Property

Statement	Property
a. a^{-2} = D, $a \neq 0$	A. Product of Powers
b. $(ab)^4$ = C	B. Power of a Power
c. $(a^3)^4$ = B	C. Power of a Product
d. $a^3 \bullet a^4$ = A	D. Negative Exponent
e. $\left(\frac{a}{b}\right)^3$ = G, $b \neq 0$	E. Zero Exponent
f. $\frac{a^6}{a^2}$ = F, $a \neq 0$	F. Quotient of Powers
g. a^0 = E, $a \neq 0$	G. Power of a Quotient

Exploration 1

Product rules are shown again on slide #6 with book definitions and with examples.

Work with a partner. Show that you can apply the properties of integer exponents to rational exponents by simplifying each expression. Use a calculator to check your answers.

a. $5^{2/3} \cdot 5^{4/3}$ **b.** $3^{1/5} \cdot 3^{4/5}$ **c.** $(4^{2/3})^3$

d. $(10^{1/2})^4$ **e.** $\frac{8^{5/2}}{8^{1/2}}$ **f.** $\frac{7^{2/3}}{7^{5/3}}$

Exploration 2

a.) bases are the same, add and simplify exponents.
b.) same as a
c.) power to power - multiply and simplify the exponents
d.) same as c
E + F.) Bases are the same, subtract and simplify the exponents.

Work with a partner. Use the properties of exponents to write each expression as a single radical. Then evaluate each expression. Use a calculator to check your answers.

a. $\sqrt{3} \cdot \sqrt{12}$ **b.** $\sqrt[3]{5} \cdot \sqrt[3]{25}$ **c.** $\sqrt[4]{27} \cdot \sqrt[4]{3}$

d. $\frac{\sqrt{98}}{\sqrt{2}}$ **e.** $\frac{\sqrt[4]{4}}{\sqrt[4]{1024}}$ **f.** $\frac{\sqrt[3]{625}}{\sqrt[3]{5}}$

Exploration 3

Multiplication: multiply both values and put under the radical and simplify when you can.

Division: put all values under one radical and simplify the fraction as you can making sure there are no radicals left in the denominator.

Core Concept

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Core Concept

*Important to understand And be able to use

*Can keep a copy in notes to use on tests and Quizzes

Use the properties of rational exponents to simplify each expression.

a. $7^{1/4} \bullet 7^{1/2} = 7^{(1/4+1/2)} = 7^{3/4}$

b. $\left(6^{1/2} \bullet 4^{1/3}\right)^2 = \left(6^{1/2}\right)^2 \bullet \left(4^{1/3}\right)^2 = 6^{(1/2 \bullet 2)} \bullet 4^{(1/3 \bullet 2)} = 6^1 \bullet 4^{2/3} = 6 \bullet 4^{2/3}$

c. $\left(4^5 \bullet 3^5\right)^{-1/5} = \left[(4 \bullet 3)^5\right]^{-1/5} = \left(12^5\right)^{-1/5} = 12^{[5 \bullet (-1/5)]} = 12^{-1} = \frac{1}{12}$

Example 1 a-c

*Steps are done out on this slide - remember that when adding or subtracting fractions, the denominators must be the same (common) If they are not the same you make them the same by multiplying by the same number on the top and bottom (numerator and denominator.)

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} = 5^{(1-1/3)} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\frac{42}{6}\right)^{1/3}\right]^2 = \left(7^{1/3}\right)^2 = 7^{(1/3 \bullet 2)} = 7^{2/3}$

Example 1 d-e

example e: Some times it is easier to use the property rules in reverse. It will take fewer steps and less chance for mistakes.

Simplify the expression.

1. $2^{3/4} \bullet 2^{1/2}$

2. $\frac{3}{3^{1/4}}$

3. $\left(\frac{20^{1/2}}{5^{1/2}}\right)^3$

4. $\left(5^{1/3} \bullet 7^{1/4}\right)^3$

Monitoring Progress 1-4

practice problems - ask if questions

Core Concept

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

Core Concept

· additional definitions that can be used on tests and Quizzes.

Use the properties of radicals to simplify each expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ Product Property of Radicals

b. $\frac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\frac{80}{5}} = \sqrt[4]{16} = 2$ Quotient Property of Radicals

An expression involving a radical with index n is in **simplest form** when these three conditions are met.

- No radicands have perfect nth powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

To meet the last two conditions, rationalize the denominator. This involves multiplying both the numerator and denominator by an appropriate form of 1 that creates a perfect nth power in the denominator.

Example 2

· Always make sure to simplify the complete problem

· No exponents left as fractions

· No radicals left in the denominator

Write each expression in simplest form.

a. $\sqrt[3]{135}$

5 27

3 9

3 3

$\boxed{3\sqrt[3]{5}}$

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7} \cdot \sqrt[5]{2 \cdot 2}}{\sqrt[5]{2 \cdot 2 \cdot 2}(\sqrt[5]{2 \cdot 2})}$

$\boxed{\frac{\sqrt[5]{28}}{2}}$

Example 3

* don't use calculator to simplify - credit will not be given if a calculator is used.

Write $\frac{1}{5+\sqrt{3}}$ in simplest form.

$$\frac{1}{5+\sqrt{3}}\cdot\frac{(5-\sqrt{3})}{(5-\sqrt{3})}=\frac{5-\sqrt{3}}{22}$$

	5	$+\sqrt{3}$
5	25	$5\sqrt{3}$
$-\sqrt{3}$	$-5\sqrt{3}$	$-\sqrt{9}=-3$

$=25-3=22$

Example 4

Rationalize the denominator use the same terms, but opposite operations - the binomials will be the factored form of the difference of perfect squares.

Simplify each expression.

a. $\sqrt[4]{10}+7\sqrt[4]{10}$ b. $2(8^{1/5})+10(8^{1/5})$ c. $\sqrt[3]{54}-\sqrt[3]{2}$

$1\sqrt[4]{10}+7\sqrt[4]{10}$

$\boxed{8\sqrt[4]{10}}$

b) $2\sqrt[5]{8}+10\sqrt[5]{8}$

$\boxed{12\sqrt[5]{8}}$

c) $3\sqrt[3]{2}-\sqrt[3]{2}$

$\boxed{2\sqrt[3]{2}}$

54
2 27
3 9
3 3

Example 5

a) when index and number under the radical are the same, add the coefficient

b) switch rational exponents to radicals the add.

c) Simplify each radical then subtract.

Simplify the expression.

5. $\sqrt[4]{27}\bullet\sqrt[4]{3}$ 6. $\frac{\sqrt[3]{250}}{\sqrt[3]{2}}$ 7. $\sqrt[3]{104}$ 8. $\sqrt[5]{\frac{3}{4}}$

9. $\frac{3}{6-\sqrt{2}}$ 10. $7\sqrt[5]{12}-\sqrt[5]{12}$ 11. $4(9^{2/3})+8(9^{2/3})$ 12. $\sqrt[3]{5}+\sqrt[3]{40}$

Monitoring Progress 5-12

* additional practice - do on your own, please ask if you have questions.

Simplify each expression.

a. $\sqrt[3]{64y^6}$

4 16
44

y·y·y
y·y·y

$\sqrt[3]{64y^6} =$

$\boxed{4y^2}$

b. $\sqrt[4]{\frac{x^4}{y^8}}$

$\frac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}}$ $\to \frac{4}{4} = 1$, $\to \frac{8}{4} = 2$

$\boxed{\frac{x}{y^2}}$

Example 6

follow the same steps even when variables are used.

* when exponents are under the radical, use the Index and divide the exponent, that is the value you take out from under the radical

Write each expression in simplest form.
Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5}$ b. $\frac{x}{\sqrt[3]{y^8}}$ c. $\frac{14xy^{4/3}}{2x^{3/4}z^{-6}}$

Example 7

* Student practice - please try on your own and ask any questions

Perform each indicated operation.
Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y}$ b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

Example 8

* Student practice - please try on your own And ask any questions.

Simplify the expression. Assume all variables are positive.

13. $\sqrt[3]{27q^9}$

14. $\sqrt[5]{\frac{x^{10}}{y^5}}$

15. $\frac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$

16. $\sqrt{9w^5} - w\sqrt{w^3}$

Monitoring Progress 13-16

* additional student practice.

• **Muddiest Point:** Ask students to identify, aloud or on a paper to be collected, the muddiest point(s) about the lesson. What was difficult to understand?

Closure

*Send me an e-mail with anything that you still need help on. Please be specific. Check to see when I am after school so we can meet to go over questions.