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5.1 *n*th Roots and Rational Exponents

For use with Exploration 5.1

Essential Question How can you use a rational exponent to represent a power involving a radical?

1 EXPLORATION: Exploring the Definition of a Rational Exponent

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use a calculator to show that each statement is true.

a. $\sqrt{9} = 9^{1/2}$ **b.** $\sqrt{2} = 2^{1/2}$ **c.** $\sqrt[3]{8} = 8^{1/3}$

d. $\sqrt[3]{3} = 3^{1/3}$ **e.** $\sqrt[4]{16} = 16^{1/4}$ **f.** $\sqrt[4]{12} = 12^{1/4}$

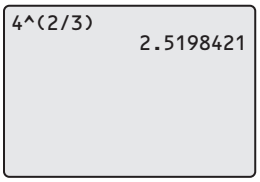
2 EXPLORATION: Writing Expressions in Rational Exponent Form

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the definition of a rational exponent and the properties of exponents to write each expression as a base with a single rational exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample

$$\left(\sqrt[3]{4}\right)^2 = \left(4^{1/3}\right)^2$$
$$= 4^{2/3}$$
$$\approx 2.52$$



a. $\left(\sqrt{5}\right)^3$ **b.** $\left(\sqrt[4]{4}\right)^2$ **c.** $\left(\sqrt[3]{9}\right)^2$

d. $\left(\sqrt[5]{10}\right)^4$ **e.** $\left(\sqrt{15}\right)^3$ **f.** $\left(\sqrt[3]{27}\right)^4$

3 EXPLORATION: Writing Expressions in Radical Form

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the properties of exponents and the definition of a rational exponent to write each expression as a radical raised to an exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample $5^{2/3} = \left(5^{1/3}\right)^2 = \left(\sqrt[3]{5}\right)^2 \approx 2.92$

a. $8^{2/3}$ **b.** $6^{5/2}$ **c.** $12^{3/4}$

d. $10^{3/2}$ **e.** $16^{3/2}$ **f.** $20^{6/5}$

Communicate Your Answer

4. How can you use a rational exponent to represent a power involving a radical?

5. Evaluate each expression *without* using a calculator. Explain your reasoning.

a. $4^{3/2}$ **b.** $32^{4/5}$ **c.** $625^{3/4}$

d. $49^{3/2}$ **e.** $125^{4/3}$ **f.** $100^{6/3}$

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5.1 Notetaking with Vocabulary

For use after Lesson 5.1

In your own words, write the meaning of each vocabulary term.

*n*th root of *a*

index of a radical

Core Concepts

Real *n*th roots of *a*

Let n be an integer $(n > 1)$ and let a be a real number.

	***n* is an even integer.**	***n* is an odd integer.**
$a < 0$	No real nth roots	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

Notes:

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5.1 Notetaking with Vocabulary (continued)

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}, a \neq 0$$

Notes:

Extra Practice

In Exercises 1–3, find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = -125$ **2.** $n = 2, a = -400$ **3.** $n = 6, a = 64$

In Exercises 4–11, evaluate the expression without using a calculator.

4. $64^{1/2}$ **5.** $(-27)^{1/3}$ **6.** $32^{7/5}$ **7.** $49^{-3/2}$

8. $(-32)^{3/5}$ **9.** $1000^{-2/3}$ **10.** $81^{3/4}$ **11.** $625^{1/4}$

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5.1 Notetaking with Vocabulary (continued)

In Exercises 12–15, match the equivalent expressions. Explain your reasoning.

12. $\left(\sqrt{a}\right)^3$

13. $-\sqrt[3]{a}$

14. $\left(\sqrt[3]{a}\right)^2$

15. $\dfrac{1}{\sqrt[3]{a}}$

A. $a^{-1/3}$

B. $a^{2/3}$

C. $a^{3/2}$

D. $-a^{1/3}$

In Exercises 16–19, find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

16. $6x^3 = -6$

17. $2(x + 5)^4 = 128$

18. $x^5 - 32 = -64$

19. $-\frac{1}{10}x^3 + 100 = 0$

20. The volume of a cube is 1728 cubic inches. What are the dimensions of the cube?