Simplify.

1. $k(k^4)$

$\boxed{k^5}$

2. $(4u^5v)(6u^5v^2)$

$\boxed{24u^{10}v^3}$

3. $(5a^3b^{10}c)^2$

$\boxed{25a^6 10^2 c^2}$

4. $(3x^3y)(3xy^2z)^4(3xyz)$

$3x^3y \cdot 81x^4y^8z^4 \cdot 3xyz$

$\boxed{729x^8y^{10}z^5}$

5. $(-g^2h)(-2gj^3)^3(-ghj)^4$

$-g^2 \cdot h \cdot -8 \cdot g^3j^9 \cdot -g^4h^4j^4$

$\boxed{-8g^9h^5j^{13}}$

6. $(2xy^5)(-y)^4$

$2 \cdot x \cdot y^5 \cdot 1 \cdot y^4$

$\boxed{2xy^9}$

Warm Up

Warm up: remember your exponent rules: you will need them in this section and in future sections.
• You can create a list for your notes to use on tests and quizzes.

Essential Question

How can you use a rational exponent to represent a power involving a radical?

exponent

$\left(\text{index} \rightarrow \sqrt{\quad} \leftarrow \text{radicand}\right)^x$

Index: a number that tells us how many roots we need

Essential Question

In this section we learn how to turn a rational exponents (an exponent that looks like a fraction) into a radical and a radical into a rational exponent

radicand: the number under the radical sign.

Work with a partner. Use a calculator to show that each statement is true.

a. $\sqrt{9} = 9^{1/2}$

b. $\sqrt{2} = 2^{1/2}$

c. $\sqrt[3]{8} = 8^{1/3}$

d. $\sqrt[3]{3} = 3^{1/3}$

e. $\sqrt[4]{16} = 16^{1/4}$

f. $\sqrt[4]{12} = 12^{1/4}$

Exploration 1

* use calculator or desmos.com to calculate

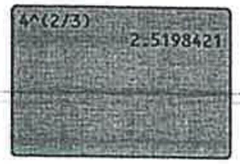
a^b - In desmos allows you to enter any exponent

function - misc. to find $\sqrt[n]{}$ allows you to enter a different index.

Work with a partner. Use the definition of a rational exponent and the properties of exponents to write each expression as a base with a single rational exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample

$$\left(\sqrt[3]{4}\right)^2 = \left(4^{1/3}\right)^2$$
$$= 4^{2/3}$$
$$\approx 2.52$$



a. $\left(\sqrt{5}\right)^3$ b. $\left(\sqrt[4]{4}\right)^2$ c. $\left(\sqrt[3]{9}\right)^2$

d. $\left(\sqrt[3]{10}\right)^4$ e. $\left(\sqrt{15}\right)^3$ f. $\left(\sqrt[3]{27}\right)^4$

Exploration 2

* Always check directions

* use a calculator
  - enter the problem as it appears on the sheet. Use parenthesis and symbols exactly as they appear.

Work with a partner. Use the properties of exponents and the definition of a rational exponent to write each expression as a radical raised to an exponent. Then use a calculator to evaluate each expression. Round your answer to two decimal places.

Sample $5^{2/3} = \left(5^{1/3}\right)^2 = \left(\sqrt[3]{5}\right)^2 \approx 2.92$

a. $8^{2/3}$ b. $6^{5/2}$ c. $12^{3/4}$

d. $10^{3/2}$ e. $16^{3/2}$ f. $20^{6/5}$

Exploration 3

Additional Student practice

Core Concept

Real nth Roots of a

Let n be an integer ($n > 1$) and let a be a real number.

n is an even integer.		n is an odd integer.	
$a < 0$	No real nth roots	$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

Core Concept

* notes on the difference between even roots and odd roots → * watch your signs.

Find the indicated real nth root(s) of a.

a. $n = 3$, $a = -216$

$\sqrt[n]{a}$

$\sqrt[3]{-216}$

-6

b. $n = 4$, $a = 81$

$\sqrt[n]{a}$

$\sqrt[4]{81}$

3

Example 1

n = root
a = radicand

Set up and simplify

Find the indicated real nth root(s) of a.

1. $n = 4$, $a = 16$
2. $n = 2$, $a = -49$
3. $n = 3$, $a = -125$
4. $n = 5$, $a = 243$

Monitoring Progress 1-4

*Student practice

Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$$

Core Concept 2

a becomes the radicand
n (denominator) becomes the Index
m (numerator/exponent) becomes the exponents

negative exponent: Switch location - If begins on top put to bottom. If begins on bottom put on top - negative exponent will become positive.

Evaluate each expression.

a. $16^{3/2}$

$(\sqrt[2]{16})^3$

4^3

$\boxed{64}$

b. $32^{-3/5}$

$\frac{1}{32^{3/5}}$

$\frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3}$

$\boxed{\frac{1}{8}}$

Example 2

Steps to simplify
a) turn from rational exponent to radical and simplify
b) get exponent positive, then turn from rational exponent to radical and simplify - don't drop the fraction.

Evaluate each expression using a calculator. Round your answer to two decimal places.

a. $9^{1/5}$ b. $12^{3/8}$ c. $(\sqrt[4]{7})^3$

Example 3

use a calculator. * Student practice.

Evaluate the expression without using a calculator.

5. $4^{5/2}$ 6. $9^{-1/2}$ 7. $81^{3/4}$ 8. $1^{7/8}$

Evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

9. $6^{2/5}$ 10. $64^{-2/3}$ 11. $(\sqrt[4]{16})^5$ 12. $(\sqrt[3]{-30})^2$

Monitoring Progress 5-12

* Student practice
-use calculator

Find the real solution(s) of (a) $4x^5 = 128$ and (b) $(x - 3)^4 = 21$.

$\frac{4x^5}{4} = \frac{128}{4}$

$x^5 = 32$

$\sqrt[5]{x^5} = \sqrt[5]{32}$

$x = 2$

$(x-3)^4 = 21$

$\sqrt[4]{x-3} = \sqrt[4]{21}$

$x - 3 = \pm 2.14$

$x - 3 = 2.14$ $\quad$ $x - 3 = -2.14$

$+3 \quad +3$ $\quad$ $+3 \quad +3$

$x \approx 5.14$ $\quad$ $x \approx .86$

Example 4

- Isolate the variable on one side then solve.

In b, solve for positive and negative because it is an even root
- two answers

* when rounding we should change = to ≈ because we approximate the solution

A hospital purchases an ultrasound machine for $50,000. The hospital expects the useful life of the machine to be 10 years, at which time its value will have depreciated to $8000. The hospital uses the declining balances method for depreciation, so the annual depreciation rate r (in decimal form) is given by the formula

$$r = 1 - \left(\frac{S}{C}\right)^{1/n}.$$

In the formula, n is the useful life of the item (in years), S is the salvage value (in dollars), and C is the original cost (in dollars). What annual depreciation rate did the hospital use?

$r = 1 - \left(\frac{8{,}000}{50{,}000}\right)^{1/10} = 1 - \left(\frac{4}{25}\right)^{1/10}$

$r \approx 0.167$

Example 5

S = 8,000
C = 50,000
n = 10 years

The annual depreciation rate is about 0.167 or 16.7%

Find the real solution(s) of the equation. Round your answer to two decimal places when appropriate.

13. $8x^3 = 64$ $\quad$ **14.** $\frac{1}{2}x^5 = 512$

15. $(x + 5)^4 = 16$ $\quad$ **16.** $(x - 2)^3 = -14$

17. WHAT IF? In Example 5, what is the annual depreciation rate when the salvage value is $6000?

Monitoring Progress 13-17

* Student practice

• Exit Ticket: Evaluate $49^{3/2}$ without a calculator.

$(\sqrt[2]{49})^3 = 7^3 = 343$

Evaluate $(\sqrt[3]{-24})^2$ with a calculator.

$(\sqrt[3]{-24})^2 = 8.32$

Closure

Additional practice -

make sure to reach out with questions if you have them.