* Warm up* Spiral Review

Solve.

1. $4t - 6 = -7$
$+6 \quad +6$
$4t = -1$
$t = -1/4$

2. $8x - 28 = -68$
$+28 \quad +28$
$8x = -40$
$x = -5$

3. $-3 - \frac{r}{10} = 3$
$+3 \quad +3$
$-10 \cdot -\frac{r}{10} = 6 \cdot -10$
$r = -60$

4. $15 = \frac{5-z}{-1} \cdot -1$
$-15 = 5 - z$
$-5 \quad -5$
$-z = -20$
$z = 20$

5. $-7m + 9 = 23$
$-9 \quad -9$
$-7m = 14$
$m = -2$

6. $\frac{3b}{2} - 7 = 23$
$+7 \quad +7$
$2 \cdot \frac{3b}{2} = 30 \cdot 2$
$3b = 60$
$b = 20$

Warm Up

• Solve for the Variable → Combine like terms and follow order of operations

• remember Isolate the variable !

• What does Isolate mean

Solve the inequality algebraically.

1. $5x^2 > 25$
$x^2 > 5$
$x > \pm\sqrt{5}$

2. $x^2 + 12x \leq -27$
$x^2 + 12x + 27 \leq 0$
$(x+3)(x+9) \leq 0$
$x \leq -3 \quad x \leq -9$

3. $x^2 + 6x + 6 \geq 1$
$x^2 + 6x + 5 \geq 0$
$(x+1)(x+5) \geq 0$
$x \geq -1 \quad x \geq -5$

4. $x^2 < 5$

5. $x^2 + 2x - 4 > -1$

6. $x^2 - 9x < -8$

Cumulative Warm Up

27
1 27
$\boxed{3 \quad 9}$

Essential Question

How can you determine whether a polynomial equation has a repeated solution?

Zeros
Roots
Solutions

how are they related?

Do they mean the same thing?

Essential Question

What you will learn:

• Find solutions to polynomials equations and Zeros of polynomial functions
• Use the rational Root theorem
• Use Irrational conjugates theorem

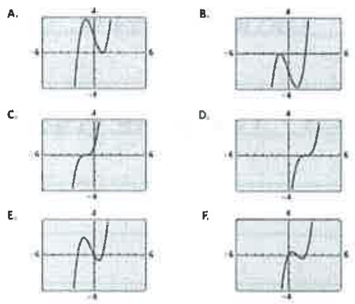
2

Work with a partner. Some cubic equations have three distinct solutions. Others have repeated solutions. Match each cubic polynomial equation with the graph of its related polynomial function. Then solve each equation. For those equations that have repeated solutions, describe the behavior of the related function near the repeated zero using the graph or a table of values.

a. $x^3 - 6x^2 + 12x - 8 = 0$ **b.** $x^3 + 3x^2 + 3x + 1 = 0$

c. $x^3 - 3x + 2 = 0$ **d.** $x^3 + x^2 - 2x = 0$

e. $x^3 - 3x - 2 = 0$ **f.** $x^3 - 3x^2 + 2x = 0$



Exploration 1

a → D $x = 2$ $x = 2$ $x = 2$

b → C $x = -1$ $x = -1$ $x = -1$

c → A $x = -2$ $x = 1$ $x = 1$

d → E $x = -2$ $x = 0$ $x = 1$

e → B $x = -1$ $x = -1$ $x = -1$

f → F $x = 0$ $x = 1$ $x = 2$

• Use graphing calculators to graph and match

• Explore pieces of the function that can help if we did not have a graphing calculator.

Work with a partner. Determine whether each quartic equation has repeated solutions using the graph of the related quartic function or a table of values. Explain your reasoning. Then solve each equation.

a. $x^4 - 4x^3 + 5x^2 - 2x = 0$

repeated solutions

$x = 0$ $x = 1$ $x = 1$ $x = 2$

b. $x^4 - 2x^3 - x^2 + 2x = 0$

∅ repeats

$x = -1$ $x = 0$

$x = 0$

~~$x =$~~

c. $x^4 - 4x^3 + 4x^2 = 0$

2 repeated solutions

d. $x^4 + 3x^3 = 0$

repeated solutions

Exploration 2

• Work with a partner (can use a graphing calculator)

• Students discuss what is happening with each function

Solve $2x^3 - 12x^2 + 18x = 0$.

$2x(x^2 - 6x + 9) = 0$ GCF $2x$

$2x(x-3)(x-3) = 0$ factor polynomial

$2x(x-3)^2 = 0$ perf. sq. trinomial

$2x = 0$ $x - 3 = 0$ zero product property

$x = 0$ $x = 3$

Solutions $x = 0$, $x = 3$

Example 1

• Always look for a GCF → Greatest common factor

• Remember a GCF only happens when that value is in every term

• What is a term?

• Use technology to graph and show solutions.

Find the zeros of $f(x) = -2x^4 + 16x^2 - 32$. Then sketch a graph of the function.

$-2(x^4 - 8x^2 + 16) = 0$

$-2(x^2 - 4)(x^2 - 4) = 0$

$-2(x-2)(x+2)(x-2)(x+2) = 0$

$-2(x+2)^2(x-2)^2 = 0$

$x+2=0$ $\quad$ $x-2=0$

$x=-2$ $\quad$ $x=2$

• b/c even exponent ⇒ graph will touch @ the zeros, not cross.

Example 2

• Begin by checking for a GCF

• f(x) = y to find the zero we look for the x-intercepts. *

• x-intercepts are found when y = 0

• Use technology to graph and review graph compared to polynomial

must show work

Solve the equation.

1. $4x^4 - 40x^2 + 36 = 0$

2. $2x^5 + 24x = 14x^3$

$4(x^4 - 10x^2 + 9) = 0$

$4(x^2-9)(x^2-1) = 0$

$4(x+3)(x-3)(x-1)(x+1) = 0$

Find the zeros of the function. Then sketch a graph of the function. with technology

Show work to solve

3. $f(x) = 3x^4 - 6x^2 + 3$

4. $f(x) = x^3 + x^2 - 6x$

Monitoring Progress 1-4

additional practice

1.) $x = -3$ $\quad$ $x = 1$

$x = 3$ $\quad$ $x = -1$

Core Concept

The Rational Root Theorem

If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Core Concept

• Rational Root theorem can be a starting point for finding solutions of a polynomial equation. However, the theorem lists only possible solutions. In order to find the actual solutions, you must test values from the list of possible solutions.

Find all real solutions of $x^3 - 8x^2 + 11x + 20 = 0$.

LC = 1 Constant $\underline{20}$

1 20
2 10
-4 -5

List $\frac{P}{Q} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$

$$\begin{array}{r|rrr:r} 1 & 1 & -8 & 11 & 20 \\ & \downarrow & 1 & -7 & 4 \\ \hline & 1 & -7 & 4 & 24 \end{array} \qquad \begin{array}{r|rrr:r} -1 & 1 & -8 & 11 & 20 \\ & \downarrow & -1 & 9 & -20 \\ \hline & 1 & -9 & 20 & 0 \end{array}$$

$f(1) \neq 0$ — $x-1$ is not a factor

$f(-1) = 0$ — $x+1$ is a factor

Example 3

- Not easily factorable - use Rational root theorem
- list all possible solutions
  p = factors of constant term
  q = factor of leading coefficient
- Test possible solutions using Synthetic Division
- factor completely:
  $x^3 - 8x^2 + 11x + 20 = 0$
  $(x+1)(x^2 - 9x + 20) = 0$
  $(x+1)(x-4)(x-5) = 0$
- → $x = -1$ $x = 4$ $x = 5$

Find all real zeros of $f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$.

Q = LC = $\underline{10}$ P = constant = $\underline{12}$

1 10
2 5

1 12
2 6
3 4

Possible Solutions: $\frac{P}{Q}$

$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{3}{2}$

$\pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}$

$\pm\frac{1}{10}, \pm\frac{3}{10}$

Example 4

*when $\frac{P}{Q}$ repeat only list once

- because so many solutions graph w/ technology to find reasonable solutions

$x = -3/2$ $x = -1/2$ $x = 3/5$
$x = 12/5$

use textbook to see the factored polynomial (pg. 192)

5. Find all real solutions of $x^3 - 5x^2 - 2x + 24 = 0$.

6. Find all real zeros of $f(x) = 3x^4 - 2x^3 - 37x^2 + 24x + 12$.

Monitoring Progress 5-6

Student practice

* practice listing out all possible solutions

* practice synthetic division.

Core Concept

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

Core Concept

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 3 and $2 + \sqrt{5}$.

Example 5

7. Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 4 and $1 - \sqrt{5}$.

Monitoring Progress 7

Exit Ticket: Given that $\frac{1}{2}$ is a zero of $f(x) = 48x^3 + 4x^2 - 20x + 3$, find the remaining zeros.

Closure