Chapter 4 Section 2 Notes

Work with a partner. Find each product. Show your steps.

a. $(x+1)^3 = (x+1)(x+1)^2$ Rewrite as a product of first and second powers.
$= (x+1)$ ______ Multiply second power.
$=$ ______ Multiply binomial and trinomial.
$=$ ______ Write in standard form, $ax^3 + bx^2 + cx + d$.

b. $(a+b)^3 = (a+b)(a+b)^2$ Rewrite as a product of first and second powers.
$= (a+b)$ ______ Multiply second power.
$=$ ______ Multiply binomial and trinomial.
$=$ ______ Write in standard form.

Exploration 1a-b

· walk through process

$(x+1)^3 =$

$(x+1)(x+1)(x+1)$

$(x^2+2x+1)(x+1)$

$x^3+x^2+2x^2+2x+x+1$

x^3+3x^2+3x+1

Work with a partner. Find each product. Show your steps.

c. $(x-1)^3 = (x-1)(x-1)^2$ Rewrite as a product of first and second powers.
$= (x-1)$ ______ Multiply second power.
$=$ ______ Multiply binomial and trinomial.
$=$ ______ Write in standard form.

d. $(a-b)^3 = (a-b)(a-b)^2$ Rewrite as a product of first and second powers.
$= (a-b)$ ______ Multiply second power.
$=$ ______ Multiply binomial and trinomial.
$=$ ______ Write in standard form.

Exploration 1c-d

$(x-1)^3$

$(x-1)(x-1)(x-1)$

$(x^2-x-x+1)(x-1)$

$(x^2-2x+1)(x-1)$

$x^3-x^2-2x^2-2x+1x-1$

x^3-3x^2-x-1

Work with a partner.

a. Use the results of Exploration 1 to describe a pattern for the coefficients of the terms when you expand the cube of a binomial. How is your pattern related to Pascal's Triangle, shown at the right?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

b. Use the results of Exploration 1 to describe a pattern for the exponents of the terms in the expansion of a cube of a binomial.

c. Explain how you can use the patterns you described in parts (a) and (b) to find the product $(2x-3)^3$. Then find this product.

Exploration 2

Simplify.

1. (5 − 4)7
   (1) 7
   7
2. −1(x − 7)
   −x + 7
3. (3 + 4m)7
   21 + 28m
4. 14r − 4r
   10r

typo → 5. 6z2 − 2z − 9z2
   2nd powers
   $6z^2 - 2z - 9z^2$
   $-3z^2 - 2z$
6. 6m − 3m − 4 p − 5m
   3m − 4p − 5m
   −2m − 4p

Warm Up

* Review of distributive property

· Simplify when possible

· Combine like terms - Must have same variables and exponents in order to combine coefficients

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 5x + c$
   $\frac{5}{2}$ $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$
   $(x + 5/2)^2$
2. $z^2 + 6z + c = 9$
   $\frac{6}{2} = 3$ $(z + 3)^2$
   $3^2 = 9$
3. $w^2 - 12w + c = 36$
   $\frac{12}{2} = 6$ $(w - 6)^2$
   $6^2 = 36$
4. $x^2 - 25x + c = \frac{625}{4}$
   $\left(\frac{25}{2}\right)^2$ $625/4$ $(x - 25/2)^2$
5. $x^2 - 8x + c = 16$
   $\frac{8}{2} = 4$ $(x - 4)^2$
   $4^2 = 16$
6. $s^2 + 27x + c$

Cumulative Warm Up

Complete the square
· divide b term by 2
· Square the quotient
· add the answer to both sides (becomes c)

Write binomial
· Use value you squared
$ax^2 + bx + c$ $ax^2 - bx + c$
$(ax + \frac{b}{2})^2$ $(ax - b/2)^2$

Essential Question
How can you cube a binomial?

$(x - 2)^3$

$(x - 2)(x - 2)(x - 2)$

Essential Question

Review of foil

Multiple methods
· Foil
· double distributive
· area model

a. Add $3x^3 + 2x^2 - x - 7$ and $x^3 - 10x^2 + 8$ in a vertical format.

$$\begin{array}{r} 3x^3 + 2x^2 - x - 7 \\ + \underline{\;x^3 - 10x^2 + 0x + 8} \\ 4x^3 - 8x^2 - x + 1 \end{array}$$

b. Add $9y^3 + 3y^2 - 2y + 1$ and $-5y^2 + y - 4$ in a horizontal format.

$9y^3 + 3y^2 - 2y + 1 + (-5y^2) + y - 4$

$9y^3 - 2y^2 - y - 3$

Example 1

vertical: line terms up in an up and down format. Use place holders if all exponent (variables) are not represented

horizontal: write as an expression and combine like terms.

a. Subtract $2x^3 + 6x^2 - x + 1$ from $8x^3 - 3x^2 - 2x + 9$ in a vertical format.

$2x^3 + 6x^2 - x + 1$

b. Subtract $3z^2 + z - 4$ from $2z^2 + 3z$ in a horizontal format.

$2z^2 + 3z - (3z^2 + z - 4)$

$2z^2 + 3z + (-3z^2) - z + 4$

$-z^2 + 2z + 4$

Example 2

with subtraction: remember you must subtract every term in the second binomial.

* can use additive Inverse property: you can change subtraction to addition if you change the sign of each term that follows.

Find the sum or difference.

1. $(2x^2 - 6x + 5) + (7x^2 - x - 9)$

2. $(3t^3 + 8t^2 - t - 4) - (5t^3 - t^2 + 17)$

Monitoring Progress 1-2

Use either method to add or subtract

*Student practice

a. Multiply $-x^2 + 2x + 4$ and $x - 3$ in a vertical format.

b. Multiply $y + 5$ and $3y^2 - 2y + 2$ in a horizontal format.

Example 3

* Student practice

Multiply $x - 1$, $x + 4$, and $x + 5$ in a horizontal format.

$(x-1)(x+4)(x+5)$

$(x^2+4x-x-4)(x+5)$

$(x^2+3x-4)(x+5)$

$x^3+5x^2+3x^2+15x-4x-20$

$x^3+8x^2+11x-20$

Example 4

when multiplying more than 2 binomials together begin by multiplying 2 and then multiply by the third

- always combine like terms.

Core Concept

Special Product Patterns

Sum and Difference	**Example**
$(a+b)(a-b) = a^2 - b^2$	$(x+3)(x-3) = x^2 - 9$
Square of a Binomial	**Example**
$(a+b)^2 = a^2 + 2ab + b^2$	$(y+4)^2 = y^2 + 8y + 16$
$(a-b)^2 = a^2 - 2ab + b^2$	$(2t-5)^2 = 4t^2 - 20t + 25$
Cube of a Binomial	**Example**
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(z+3)^3 = z^3 + 9z^2 + 27z + 27$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(m-2)^3 = m^3 - 6m^2 + 12m - 8$

Core Concept

- Short cuts
  - always use the regular method

a. Prove the polynomial identity for the cube of a binomial representing a sum:

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

b. Use the cube of a binomial in part (a) to calculate 11^3.

$(a+b)(a+b)(a+b)$

$(a^2+2ab+b^2)(a+b)$

$a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$

$a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$a^3+3a^2b+3ab^2+b^3$

Example 5

· show linear and using area model

Find each product.

a. $(4n + 5)(4n - 5)$ **b.** $(9y - 2)^2$ **c.** $(ab + 4)^3$

	$4n$	$+5$
$4n$	$16n^2$	$20n$
-5	$-20n$	-25

$16n^2-25$

$(9y-2)(9y-2)$

$81y^2-18y-18y+2$

$81y^2-36y+2$

Example 6

multiply out using any method

$(ab+4)^3$

$(ab+4)(ab+4)(ab+4)$

$(ab^2+4ab+4ab+16)(ab+4)$

$(ab^2+8ab+16)(ab+4)$

$ab^3+8a^2b^2+16ab+4ab^2$
$+32ab+64$

$\boxed{ab^3+4ab^2+8a^2b^2+48ab+64}$

Find the product.

3. $(4x^2 + x - 5)(2x + 1)$ **4.** $(y - 2)(5y^2 + 3y - 1)$

5. $(m - 2)(m - 1)(m + 3)$ **6.** $(3t - 2)(3t + 2)$

7. $(5a + 2)^2$ **8.** $(xy - 3)^3$

Monitoring Progress 3-8

* Student practice *

9. (a) Prove the polynomial identity for the cube of a binomial representing a difference: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

(b) Use the cube of a binomial in part (a) to calculate 9^3.

$(a-b)(a-b)(a-b)$

$[a(a-b) + (-b)(a-b)](a-b)$

$(a^2 - ab - ab + b^2)(a-b)$

$(a^2 - 2ab + b^2)(a-b)$

$a(a^2 - 2ab + b^2) + (-b)(a^2 - 2ab + b^2)$

$a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$

$\rightarrow a^3 - 3a^2b + 3ab^2 - b^3$

• Show linear and area model

Monitoring Progress 9

Core Concept

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a+b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a+b)^0 =$	1	1
1st row	1	$(a+b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

In general: n^{th} row gives $(a+b)^n$

1) expansion has $n+1$ terms

2) power of a begins w/ n, decreases by 1 in each term, ends in 0

3) power of b begins w/ Ø increases by 1, ends w/ power of n.

4) the sum of the powers of each term is n.

Core Concept

Use Pascal's Triangle to expand (a) $(x - 2)^5$ and (b) $(3y + 1)^3$.

$(x-2)^5$ Row 5: 1, 5, 10, 10, 5, 1

$1x^5 + 5x^4(-2) + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + 1(-2)^5$

$x^5 - 10x^4 + 10x^3(4) + 10x^2(-8) + 5x(16) - 32$

$x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

$(3y+1)^3$

Row 3: 1, 3, 3, 1

$1(3y)^3 + 3(3y)^2(1) + 3(3y)(1)^2 + 1(1)^3$

$1(27y^3) + 3(9y^2) + 9y + 1$

$27y^3 + 27y^2 + 9y + 1$

Example 7

10. Use Pascal's Triangle to expand (a) $(z + 3)^4$ and (b) $(2t - 1)^5$.

$(z+3)^4$ Row 4: 1, 4, 6, 4, 1

$1z^4 + 4(z)^3(+3) + 6(z)^2(3)^2$
$+ 4z(3)^3 + 1(3^4)$

$z^4 + 12z^3 + 6z^2(9) + 4z(27) + 81$

$z^4 + 12z^3 + 54z^2 + 108z + 81$

$(2t-1)^5$ Row 5: 1, 5, 10, 10, 5, 1

$1(2t)^5 + 5(2t)^4(-1) + 10(2t)^3(-1)^2$
$+ 10(2t)^2(-1)^3 + 5(2t)^1(-1)^4$
$+ 1(-1)^5$

$32t^5 + {}^-5(16t^4) + 10(8t^3) + 10(4t^2)(-1)$
$+ 10t - 1$

$32t^5 - 80t^4 + 80t^3 - 40t^2 + 10t - 1$

Monitoring Progress 10

Writing Prompt: Explain why $(x + 2)^3 \neq x^3 + 8$.

exit ticket!

Closure